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Guidelines for more realistic daylight exterior conditions in energy conscious designs

Computer adaptation and examples

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Guidelines for researchers and designers

Introduction

The use of daylight in interiors provides a more agreeable and inspiring environment due to its manyfold changes and daily variations. The presence or absence of sunlight also leads to improved healthy conditions, adaptation training and productivity stimulation of interior inhabitants. In this sense daylight is one of the primary elements of the sustainable environment in all human shelters whether in the urban spaces of different scale and configuration or in the multiple-purpose building interiors.
The recent 45th IFHP World Congress [1] in its urban network theme has introduced three "interwoven" subthemes represented by three I´s, i.e. Internationalism, Initiative and Intervention. Considering the effects of daylight in a certain region and city or in a particular building and interior on the individual and temporary needs of their users, it is important to realise two S´s, i.e. Sun and Sky.
Both Sun and Sky have their world-wide or international character influencing global natural conditions with their overall characteristic changes including seasonal variations as well as daily rythms. As cities are considered now as the dominant habitat of mankind their planning and structuring is based in their natural setting and environmental conditions which crucially precondition their quality, economic and welfare performance.
Even though the sunlight and skylight phenomena are global their local characteristic differ significantly and sometimes sharply from one region or city to the next. In other words although the international standards, e.g. luminous Solar Constant and Sky luminance patterns are to be taken as basic everywhere, the local atmospheric and weather conditions have to be respected in energy and environmentally conscious building designs. In fact that is the core question of the Designer´s Initiative to respond to specific natural conditions and perspective changes in his quest for Intervention, i.e. his capacity to deal and positively influence and utilise the reality through planning and design solutions.
When defining daylight sources two different and rather opposite approaches were historically developed by daylight researchers:
A) Simple and quite abstract models of these two sources were established since the 18th century following their easy and practical application, i.e.:
- sunlight was represented by parallel beams geometrically corresponding to the section and plan projections in the insolation studies,
- skylight was represented by the simplest models of the sky hemisphere with unity uniform sky luminance. The latter model was replaced by the CIE Overcast Sky Standard in 1955 [2] which more closely simulates the gradual decrease of luminance from zenith unity to the horizon 1/3 under a dense multilayer Status cloudiness. Another extreme, the CIE Clear Sky Standard was adopted internationally in 1972 [3] and recently both were approved as ISO and CIE Standards [4].
B) Under the pressure of reality an opposite extreme approach was recently followed trying to simulate more or less exactly various instantaneous states linking sunlight and skylight conditions in an arbitrary location based on local measured or momentary satellite data [5] and complex computer models, e.g. by Perez et al [6], [7].
Both extreme approaches have several defficiencies:
- while the simplest overcast sky model is acceptable only for window design in Western and Central Europe representing minimum winter conditions, the absolutely changeable model is

unsuitable for the determination of any future conditions in the designed building while depending on satellite or measured information and lacking the relation to a common standard base and critical situations. Therefore it seemed that there is a need for a worldwide applicable standard set of typical sunlight and skylight conditions respecting the possible range of real natural cases in various climatic regions and applicable for different design purposes.
The restricted applicability of the CIE Overcast Sky Standard stirred the search fore more standard skies within the CIE Expert Committee E 3-2 (later TC 3-15) since 1963 and the International Daylight Measurement Programme (IDMP) was iniciated in 1983 [8] to collect daylight availability data in various climate regions. The latter was officially started in 1991. These activities are gradually yielding results and sounder basis for the analysis of daylight climate as well as sky luminance scans provide more informations about the sky patterns. Now the draft of a set of fifteen CIE Standard Skies is under consideration based on the Slovak-American project results [8]. The frequency of occurrence of these standard skies in a particular location is to be found now to specify the local validity and importance of particular sky types from this set.

The choice of the relevant typical sky standards

Long term regular measurements in 1 or 5-minute steps are now available for thorough statistical analysis which means more detail informations on daylight climate not only of local significance but also in a wider regional sense. In this meaning 5-year Bratislava data can be considered as representative for the whole Central European climate while Athens data are typical for the Mediterranean zone.
The application of the General Sky set in both these daylight climate zones is given due to the aim or purpose and information needs, thus:

1) **For window design** winter conditions are usually taken as critical with low solar altitudes not reaching over 25 - 35 degrees. In this respect the results of the Analysis 1 are basic with the following conclusions:

- In Central Europe the winter season is characterised by dull and dark days with prevailing overcast skies. Just under 40 % of cases recorded in Bratislava (Table 1) were in the group of three extreme overcast sky types I.1, I.2 and II.2 almost evenly distributed with the peak in I.2. This fact proves the importance and justification of the CIE Overcast Sky Standard established in Western, Northern and Central Europe as a primary standard for window design and the basic comparisonal criterion for evaluating minimum daylight illuminance levels in interiors under critical winter conditions.
- In Mediterranean countries even in wintertime the weather is quite changeable with many sunny periods as documented by the relatively high sunshine durations (5-year average over 40%) and the percentage of clear sky types with the maximum of cases in V.5 (Table 1).

2) **For window glare studies** it is sometimes necessary to determine the luminance sky pattern in a particular window orientation within a specified solid angle. In this respect are often critical sunny oriented window apertures under lower sun positions during transitional seasons, e.g.:

- in Central Europe with IV.4 and V.4 sky types,
- in Mediterranean regions with four clear sky types IV.4, V.4, V.5 and VI.5 respectively.

3) **For the calculation of absolute illuminance levels** due to lighting controls, additional artificial illumination or for estimating supplementary electric lighting needs with energy saving

Table 1 Comparison of winter conditions in Bratislava and Athens, 5-year data under sunheights 5 - 35 deg.

Sky Standard		Bratislava winter Nov. Dec. Jan. Feb				Athens winter Nov. Dec. Jan. Feb			
		Number of cases			%	Number of cases			%
Code	No.	without sun	with sun	Sum		without sun	with sun	Sum	
1	2	3	4	5	6	7	8	9	10
I.1	1	6998	62	7060	12.81	885	33	918	2.67
I.2	2	7289	39	7328	13.30	875	37	912	2.66
II.1	3	6548	56	6604	11.98	1534	65	1599	4.66
II.2	4	4159	91	4250	7.71	1977	115	2092	6.09
III.1	5	3148	120	3268	5.93	1786	212	1998	5.82
III.2	6	3124	212	3336	6.05	2177	321	2498	7.28
III.3	7	2402	453	2855	5.18	1355	767	2122	6.18
III.4	8	1506	562	2068	3.75	868	955	1823	5.31
IV.2	9	1167	661	1828	3.32	600	977	1577	4.59
IV.3	10	1009	1033	2042	3.71	478	1468	1946	5.67
IV.4	11	809	1877	2686	4.87	377	2035	2412	7.02
V.4	12	568	4792	5360	9.73	257	3235	3492	10.17
V.5	13	316	3617	3933	7.14	212	4754	4966	14.46
VI.5	14	129	1354	1483	2.69	138	2790	2928	8.53
VI.6	15	124	887	1011	1.83	199	2853	3052	8.89
Sum		39296	15816	55112	100.00	13718	20617	34335	100.00

sometimes several sky type patterns have to be tested within the window solid angle from the critical interior places. In this respect due to the different orientations of such critical windows there might be considered:

- extreme winter situatiions in Central Europe which are linked with overcast sky standards I.1 and I.2 while in sumertime deep side-lit rooms with North-oriented windows will have to be checked probably under the sky pattern V.4,
- in the Mediterranean area critical situations can be expected in North-oriented deep interiors under sky types V.4 and V.5 as the latter is the most frequent type during the whole year.

4) **For energy trade-off simulations or energy saving studies** there is a need for further research and reference year schemes to define relevant sky type changes during the whole year either based on their mutual monthly or seasonally occurrence of frequency or their correlations with the relative sunshine durations available in different locations. Of course, crucial would be more detail energy saving studies based on regular and long term measurements of L_z, D_v and G_v which should be mandatory and essential especially in regions and locations with until now unspecified daylight climate.

5) **For a simpler or more detail specification of the daylight climate** in unipurpose or multipurpose studies several approaches can be taken to overcome either the complexity of

the 15 sky type standardistion on one side or the assumption of the "unrealistic" homogeneity of standard sky types.

It is evident that some of the very unhomogeneous partly cloudy skies with a complex presence of different cloud types either can fall outside the 15 standard sky range or can be very far from its simulations by the quasi-even standard luminance distribution [9]. Thus further research could be directed towards the specification of all real sky luminance patterns following the regularly recorded cloud amount or cover in the selected groups of cloud types, probably using the advantages and sky image luminance mapping by the new CCD fish-eye camera [10]. As commented earlier by Valko [11] in the future " cloudiness could be the key in overcoming the basic weakness (of the homogeneous sky standards) and cloud parameters could yield some ´fuzziness´ indices". However, this could mean a considerable rise in the number of sky standards which is in contradiction with the trend in the design practice. This study has shown that if the L_z/D_v criterion is used only very few regularly recorded skies were outside the 15 standard sky range and that for practical purposes this set is sufficient to represent general sky conditions world-wide in respect with the proposed standardisation [12]. On the other side the overall year-round considerations and long term statistics can uncover also very seldom occurrence of some sky types in a certain locality, e.g. standards VI.5 and VI.6 as well as III.4 and IV.2 are unfrequent in Bratislava and probably in Central Europe, while in Athens all overcast skies I.1 to III.1 seldom occur during the whole year there. Furthermore as shown above for certain tasks and regions a restricted number of four skies was already proposed [13].

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Abstract of basic formulae applied in examples

The coexistence of sunlight and skylight as well as their quantitative and qualitative mixtures and changes are taken as natural but in theoretical studies have to be simplified, modelled in abstraction and separation as assumed in the theory of daylight and insolation of interiors.

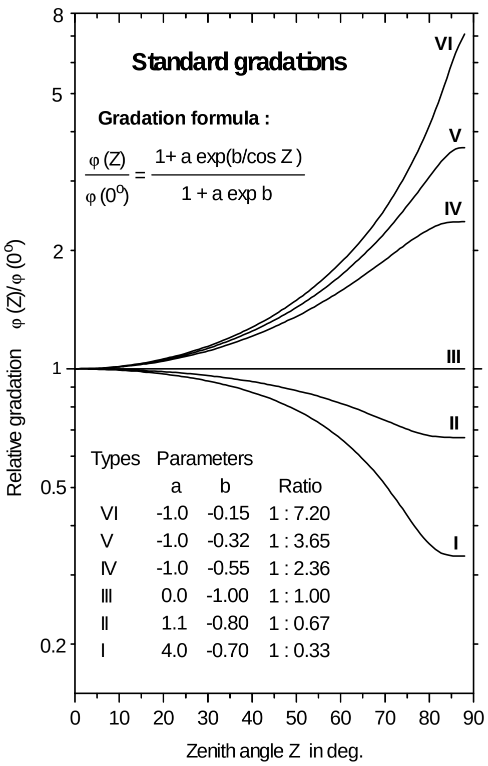


Figure 1. Standard gradation formula, parameters and profiles

The first considerations of luminance distributions were published by Lambert in his 18th Century Photometria [1]. His principle „all calculation methods are based on the assumption that the surface of the light source is of uniform (same) luminance. If this is not the case it has to be specified how luminance is changing and the influence of every element has to be multiplied by its luminance and then using the integration throughout the whole surface or its part the resulting illuminance is to be calculated“ is basis for all calculation methods.

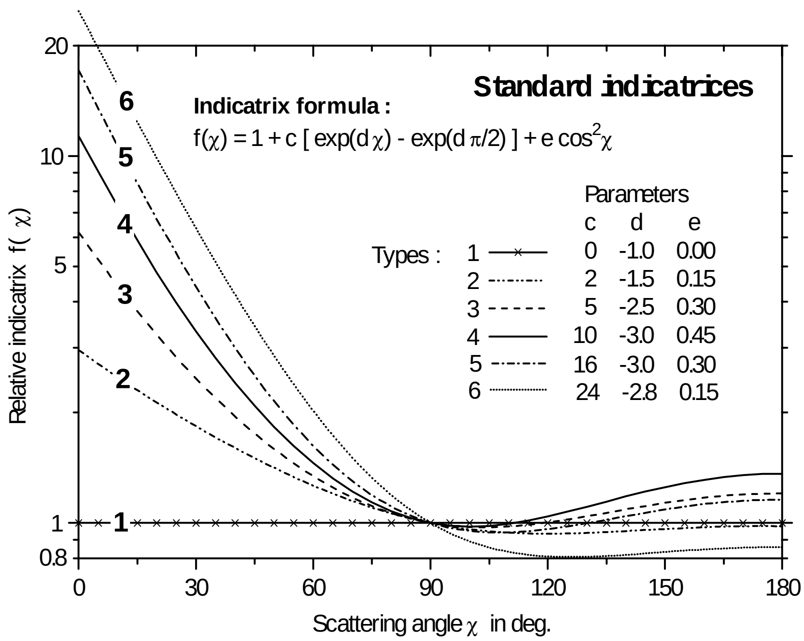
Although the uniform unity luminance of the sky hemisphere was considered a basic prerequisite and a dogmatic assumption of all daylight calculations and graphical means for a long time no proof was given whether it is only an abstract constant fiction or a really existing case. Such unity sky can represent an ideal mean linking the decreasing gradation tendency of overcast and the increasing gradation of clear skies respectively. Thus defining the new generation of sky standards it must not be forgotten to incorporate into the new set also the most abstract unity sky on which the whole knowledge of theoretical photometry is still based. In the history of daylight theory the prime aim to standardise sky patterns was either the theoretical need for a simple luminance characterisation of the sky as a large area source for calculation methods or the practical need of window design followed by its measurement check of the daylight level in real interiors. Due to the former aim the oldest and simplest sky standard was a sky of unity luminance with a constant uniformity on the whole sky vault (i.e. with a constant unity gradation and indicatrix functions), while the latter need influenced by the frequent minimum overcast conditions in Europe introduced the gradation 1:0.33 as critical. CIE has adopted this gradation for the CIE Overcast Sky Standard in 1955 [2]. An opposite gradation 1:3.65 is characteristic for very clear skies on which the brightest luminance patches are around the sun position, i.e. within the so called solar corona. Such luminance distribution was standardised in 1973 by the CIE Clear Sky Standard [3].

The gradation and indicatrix analysis have fully justified the current CIE standards for the overcast as well as the clear skies although these seem to be rather extremes of homogeneous atmospheric conditions. Furthermore the whole spectrum of gradation and indicatrix changes fully justifies also their exponential modelling which means a convenient unification and logical regularity in sky „behaviour“. Under relatively homogeneous conditions there are six relevant standard gradations represented in Figure 1 numbered by assigned Roman numerals I to VI respectively. All can be determined by appropriate *a* and *b* parameters in the formula for relative gradation:

$$\varphi(Z)/\varphi(0°) = [1 + a \exp(b/\cos Z)]/(1 + a \exp b) \qquad (1)$$

where Z is the zenith angle or the angular distance of the sky element from zenith,
$\varphi(0°)$ is the gradation function for zenith, i.e. when $Z = 0°$.
It has to be noted that the current CIE standard gradations 1:0.33, 1:1 and 1:3.65 are included in this set.

The unity indicatrix function expresses the absolute scattering uniformity of the atmosphere diffusing the incoming solar beam into all directions, i.e. a perfect Lambertian diffuser created by Mie scattering in an ideal turbid media. Under such abstract conditions the corresponding fictitious luminance solid is a sphere and its section called the indicatrix is a circle of unity radius if expressing the relative indicatrix normalised to luminance perpendicular to sun beams. This is the case of a multilayer overcast sky covered usually by a combination of cloud types including mainly Stratus cloudiness and/or fog. It is real and quite often occurring under low pressure situations in rainy or humid temperate climates.



The thinner is the air mass or less cloudy and turbid the atmosphere the less profound is the sideways directed scattering and more extended and relatively higher is the sky luminance close around the sun beam. Thus with the decreasing is gradually increasing the prolongation of the scattering indicatix in the forward direction of sunlight flow. So the distortion of the luminance solid follows from the sphere/ball shape to a pearlike form with a swelled „tail“. This transformation of the relative scattering indicatrix can be modelled by an exponential formula :

$$f(\chi) = 1 + c[\exp(d\chi) - \exp(d\pi/2)] + e \cos^2\chi \qquad (2)$$

where χ is the scattering angle, i.e. the smallest angular distance of an arbitrary sky element from the sun position which is given by the formula

$$\cos \chi = \cos Z_s \cos Z + \sin Z_s \sin Z \cos A_z \qquad (3)$$

where Z_s is the solar zenith angle and A_z is the azimuth of the Z meridian from the sun meridian.

The appropriate *c*, *d* and *e* parameters for the standard set are defined in Figure 2. Actual combinations of six standard gradations and six indicatrices can form quite many sky standards but only fifteen relevant were chosen to be contained in the new standard set [4,5] summarised in Table 2 defining the Standard Sky Luminance Distributions (SSLD). Recently the CIE Technical Committee 3-15 has recommended this set for universal standardisation [6] adopting gradation and indicatrix functions after (1) and (2) respectively as well as the luminance distributions after formula (14) in relative terms.
The inflow of sunlight reaching the outer border of Earth atmosphere in the form of parallel beams is defined by the universal quantity. It is the luminous solar constant $E_{v,c}$ = 133800 lm/m^2 or 133.8 klux if conceived like a fictitious illuminance on an imaginary plane perpendicular to momentary sun beam direction. In the same moment at any location, i.e. at ground level the illuminance level is proportional to horizontal extraterrestrial illuminance E_v, which can be computed as a function of the solar constant and the momentary solar altitude:

$$E_v = E_{vo} \sin \gamma_s = E_{vo} \cos Z_s \qquad (4)$$

where E_{vo} is the daily corrected solar constant,

$$E_{vo} = 133800\ (1 + 0.034 \cos(2\ \pi\ (J\text{-}2) / 365)) \qquad (5)$$

J is the Day number, i.e. J = 1 on 1st January to J = 365 on 31st December when February has 28 days,

γ_s is the solar altitude or Z_s is the solar zenith angle which are functionally dependent $\gamma_s = 90° - Z_s$,

$$\sin \gamma_s = \cos Z_s = \sin \varphi \sin \delta - \cos \varphi \cos \delta \cos (15°\ H) \qquad (6)$$

where φ - the geographical latitude of a particular location,

H - the number of the hour in that day.

δ - mean daily declination dependent on the number of the day within a year-J after [7]:

$$\delta = 0.006918 - 0.399912 \cos (\tau) + 0.070257 \sin(\tau) - 0.006758 \cos(2\ \tau) + 0.000907 \sin(2\ \tau) - 0.002697 \cos(3\ \tau) + 0.00148 \sin(3\ \tau) \qquad (7)$$

$$\text{where Day angle } \tau = 2\ \pi\ (J - 1)/365 \qquad (8)$$

The parallel beam/direct solar illuminance on a horizontal plane (P_v) is defined as

$$P_v = E_v \exp(-a_v\ m\ T_v) \qquad (9)$$

Table 2. A set of fifteen basic sky types/standards and their parametrisation

Code	Type of sky	Recommended or standardised parameters								
		for gradation	for indicatrix	typical D_v/E_v	*B*	*C*	*D*	*E*	for L_z in eq.(19)	typical *T*v
I.1	Overcast with the steep gradation and with azimuthal uniformity	*a*= 4 *b*= -0.7	*c*= 0 *d*= -1 *e*= 0	0.10	54.63	1.00	0.00	0.00	Because these sky standards are associated with no sunlight the relation (19) is not valid	
I.2	Overcast with the steep gradation and slight brightening toward sun	*a*= 4 *b*= -0.7	*c*= 2 *d*= -1.5 *e*= 0.15	0.18	12.35	3.68	0.59	50.47		
II.1	Overcast moderately graded with azimuthal uniformity	*a*= 1.1 *b*= -0.8	*c*= 0 *d*= -1 *e*= 0	0.15	48.30	1.00	0.00	0.00		
II.2	Overcast moderately graded and slight brightening toward sun	*a*= 1.1 *b*= -0.8	*c*= 2 *d*= -1.5 *e*= 0.15	0.22	12.23	3.57	0.57	44.27		
III.1	Overcast, foggy or cloudy with overall uniformity	*a*= 0 *b*= -1	*c*= 0 *d*= -1 *e*= 0	0.20	42.59	1.00	0.00	0.00		
III.2	Partly cloudy with a uniform gradation and slight brightening toward sun	*a*= 0 *b*= -1	*c*= 2 *d*= -1.5 *e*= 0.15	0.38	11.84	3.53	0.55	38.78		
III.3	Partly cloudy with a brighter circumsolar effect and uniform gradation	*a*= 0 *b*= -1	*c*= 5 *d*= -2.5 *e*= 0.3	0.42	21.72	4.52	0.64	34.56	*A1*= 0.957 *A2*= 1.790	12.0
III.4	Partly cloudy, rather uniform with a clear solar corona	*a*= 0 *b*= -1	*c*= 10 *d*= -3 *e*= 0.45	0.41	29.35	4.94	0.70	30.41	*A1*= 0.830 *A2*= 2.030	10.0
IV.2	Partly cloudy with a shaded sun position	*a*= -1 *b*= -0.55	*c*= 2 *d*= -1.5 *e*= 0.15	0.40	10.34	3.45	0.50	27.47	*A1*= 0.600 *A2*= 1.500	12.0
IV.3	Partly cloudy with brighter circumsolar effect	*a*= -1 *b*= -0.55	*c*= 5 *d*= -2.5 *e*= 0.3	0.36	18.41	4.27	0.63	24.04	*A1*= 0.567 *A2*= 2.610	10.0
IV.4	White - blue sky with a clear solar corona	*a*= -1 *b*= -0.55	*c*= 10 *d*= -3 *e*= 0.45	0.23	24.41	4.60	0.72	20.76	*A1*= 1.440 *A2*= -0.75	4.0
V.4	Very clear / unturbid with a clear solar corona	*a*= -1 *b*= -0.32	*c*= 10 *d*= -3 *e*= 0.45	0.10	23.00	4.43	0.74	18.52	*A1*= 1.036 *A2*= 0.710	2.5
V.5	Cloudless polluted with a broader solar corona	*a*= -1 *b*= -0.32	*c*= 16 *d*= -3 *e*= 0.3	0.28	27.45	4.61	0.76	16.59	*A1*= 1.244 *A2*= -0.84	4.5
VI.5	Cloudless turbid with a broader solar corona	*a*= -1 *b*= -0.15	*c*= 16 *d*= -3 *e*= 0.3	0.28	25.54	4.40	0.79	14.56	*A1*= 0.881 *A2*= 0.453	5.0
VI.6	White - blue turbid sky with a wide solar corona effect	*a*= -1 *b*= -0.15	*c*= 24 *d*= -2.8 *e*= 0.15	0.30	28.08	4.13	0.79	13.00	*A1*= 0.418 *A2*= 1.950	4.0

where m is the air mass penetrated and av its luminous ideal extinction, both dependent on solar altitude, while T_v is the luminous turbidity factor which approximates the number of ideally clean atmospheres representing an actual case. In general cases or calculations when there is no link to any date or daytime a yearly average solar constant is applied :

$$P_v / E_v = \exp(-a_v\, m\, T_v) \tag{10}$$

where av is luminous extinction coefficient : $a_v = \dfrac{0.1}{1 + 0.045\, m}$ (11)

while m is the optical mass : $m = \dfrac{1}{\sin \gamma_S + 0.50572\left(\gamma_S + 6.07995°\right)^{-1.6364}}$ (12)

If only global horizontal illuminance G_v and diffuse horizontal illuminance D_v are measured, then $P_v/E_v = G_v/E_v - D_v/E_v$ and the luminous turbidity T_v can be calculated as

$$T_v = \frac{-\ln P_v / E_v}{a_v\ m} \tag{13}$$

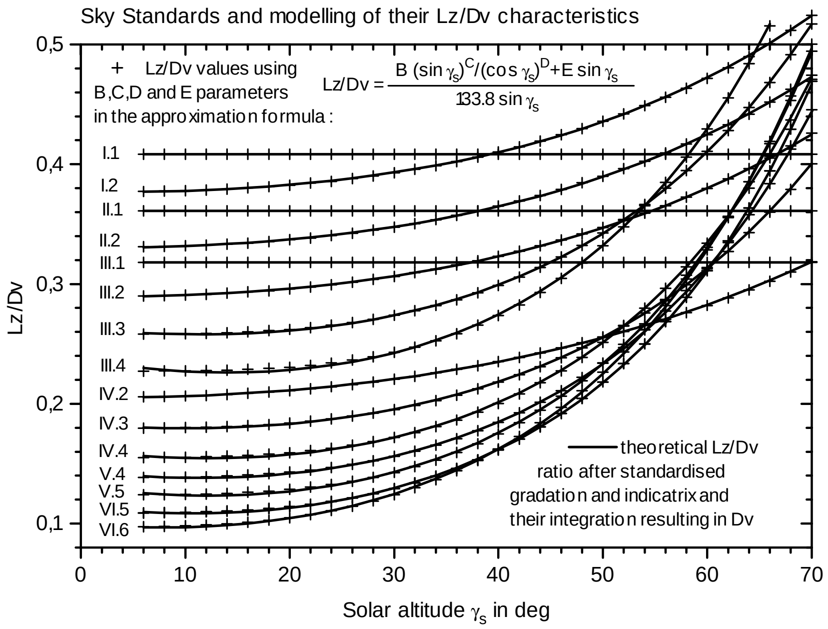


Figure 3. Courses of L_z/D_v curves for all 15 Sky Standards

The actual value of T_v represents the number of absolutely clean atmospheric filter substituting a real, e.g. polluted atmospheres having the same filtering effect on the P_v/E_v ratio under the actual solar altitude and directional air mass. Evidently the lowest T_v value is one and the highest is indefinitely high corresponding to $P_v/E_v = 0$ in the direction of sun beams.

Under arbitrary cloudy skies three parameters are essential for the case identification, i.e. global illuminance G_v or direct P_v together with simultaneous diffuse D_v and at least zenith luminance L_z have to be measures and to be interrelated in the normalised form P_v/E_v, D_v/E_v and L_z/D_v. This ´triple´ system in relation to solar altitude changes can be investigated within chosen turbidity.

The proposed new generation of sky standards is utilising the L_z/D_v properties of homogeneous skies in all 15 basic types which cover the whole spectrum of usual skies found in reality. The standardisation concept is using a twin set of gradation functions $\varphi(Z)$ and indicatrix functions $f(\chi)$ each modelled by exponential approximations by the help of *a, b, c, d* and *e* parameters, Fig. 1 and 2. The fifteen standards are formed by chosen combinations of both functions (eq. (1) and (2)) defining the relative luminance distribution for luminance L in any standard sky element as:

$$\frac{L}{L_Z} = \frac{\mathrm{f}(\chi)\,\varphi(Z)}{\mathrm{f}(Z_S)\,\varphi(0°)} \qquad (14)$$

where the ratio of gradation functions φ is defined by eq. (1), the indicatrix function f(χ) by eq. (2) and the same function when $\chi = Z_S$ is $\mathrm{f}(Z_S) = 1 + c[\exp(\mathrm{d}\,Z_S) - \exp(\mathrm{d}\pi/2)] + e\cos^2 Z_S$. Otherwise the L_Z/D_V ratio can be calculated using the integration as:

$$\frac{L_Z}{D_V} = \frac{\varphi(0°)\,\mathrm{f}(Z_S)}{\int_{Z=0}^{\pi/2}\int_{\alpha=0}^{2\pi}\left[\varphi(Z)\,\mathrm{f}(\chi)\,\sin Z\cos Z\right]\mathrm{d}Z\,\mathrm{d}\alpha} \qquad (15)$$

Now an interesting fact is evident that for each proposed standard the L_Z/D_V ratio is defined only by the angular distances of the sun and sky element from zenith and by the smallest angular distance of the sky element from sun position.

Note, that in the range of low solar altitudes the curves in Figure 3 are horizontally extended which gives the possibility of a simpler sorting routine in winter season when the sunheight is under 35°. Because the integration in the above formula (15) is tedious and complex, for practical reasons an approximation for computing L_Z/D_V can be applied using different parameters B, C, D and E for every sky standard valid for solar altitudes under 80° :

$$L_Z/D_V = 1/133.8[\,B\,(\sin\gamma_S)^C/\,(\sin\gamma_S)(\cos\gamma_S\,)^D + E] \qquad (16)$$

When the absolute zenith luminance is needed, then

$$L_Z = D_V\,/\,E_V\,[\,B\,(\sin\gamma_S)^C\,/\,(\cos\gamma_S\,)^D + E\sin\gamma_S\,] \qquad [\mathrm{kcd/m^2}] \qquad (17)$$

Note that in case of overcast skies with a unity indicatrix a constant L_Z/D_V ratio results as the value of $C = 1$, $D = E = 0$, thus eq.(16) is simplified to

$$L_Z\,/D_V = B\,/\,133.8 \qquad (18)$$

Using the computer best fit routine for all five clear sky standards their B, C, D and E parameters in eq. (12) were found for the modelling of the D_V/E_V fan. While the D_V/E_V ratio is effected mainly by the overall cloudiness and turbidity conditions, L_Z/D_V represents also the relation of the cloud influence in zenith with the turbidity in the direction of the sun beams which can be expressed by the luminous turbidity factor T_V. This at the same time indicates the filtering or shading effects of clouds on sunshine and determines the direct sunlight. So an additional approximation of L_Z as a function of T_V was found by a matching best fit for every standard sky which is expressed in a general formula valid for all intermediate and clear cases with a forward prolonged indicatrix:

$$L_Z = (A1\,T_V + A2)\sin\gamma_S + 0.7(T_V + 1)(\sin\gamma_S)^C\,/\,(\cos\gamma_S\,)^D + 0.04\,T_V \quad [\mathrm{kcd/m^2}] \qquad (19)$$

All relevant parameters associated with all Sky Standards are summarised in Table 2, where also the character of cloudiness and solar presence is indicated.

Note that in columns with typical D_v/E_v and turbidity T_v the values are suggested for orientation and practical reasons. Zenith luminance on the first six overcast and cloudy skies without sunlight have to be calculated with respect to typical D_v/E_v values when no measurements are available using equation (17) while under sunny conditions when T_v is known approximation (19) can be utilised.

References

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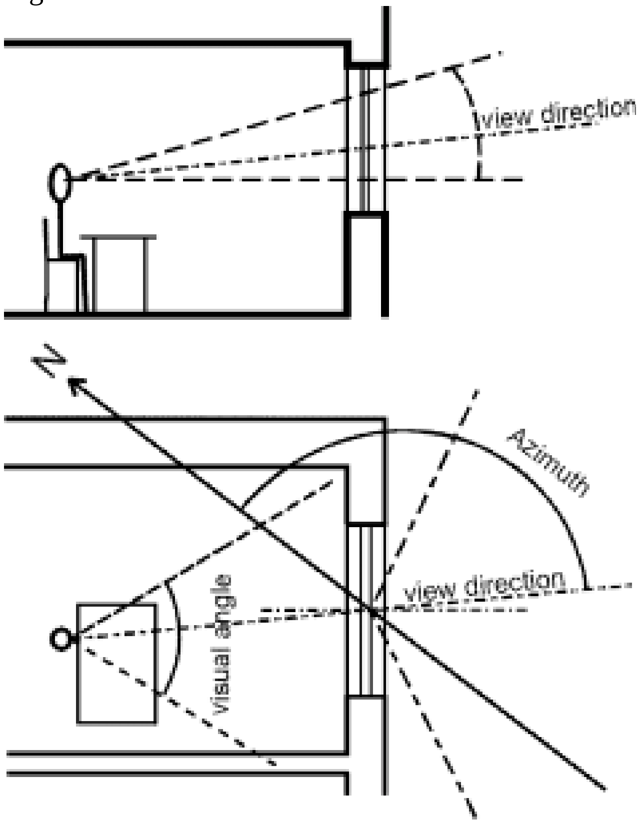
Figure E-1.1



Example 1

The office with a double glazed window is orientated to South-East. The critical view direction of the interior in habitant is in azimuth A_e=130° from North with vertical elevation γ = 10°. Due to light refraction the effective angle is ± 65 degree from normal direction of the window glass. The working time starts at 8:00.
What window luminance is the most frequent ?

Calculation procedure :

1. The exterior conditions are drawn IDMP Bratislava station during 1994 - 1996 years when the most frequent Sky types after statistical analysis are II.1 for overcast situations, III.2 for cloudy and IV.4 for clear sky situations after Tab. E-1.1.

2. The luminance distribution on the sky is defined by parameters *a, b, c, d, e.*

Tab. E-1.2 Sky type parameters for calculation of the most frequent sky luminance distribution

Sky type/ Parameters	*a*	*b*	*c*	*d*	*e*
II.1	1.10	-0.80	0.00	-1.00	0.00
III.2	0.00	-1.00	2.00	-1.50	0.15
IV.4	-1.00	-0.55	10.00	-3.00	0.45

Their values are presented in Tab. E-1.2.

3. Calculation of relevant angles.

View angle from zenith $Z = 90° - \gamma$

$Z = 90 - 10 = 80°$

Sun zenith angle $Z_s = 90° - \gamma_s$

Sun azimuth from the view direction

$A_z = | A_s - A_e |$

For Sky Type IV.4 is :

Tab. E-1.1 Results of average luminance in the relevant view angle after the statistical analysis of 5-minute Bratislava data 1994 - 1998

Type	Occurrence	Solar altitude	Solar azimuth	L_{vz}
	%	γ_s in deg	A_s in deg	cd/m2
I.1	9.41	34.28	158.275	6121
I.2	11.87	23.92	154.155	3907
II.1	14.29	30.04	158.174	5183
II.2	8.95	31.54	157.595	5780
III.1	6.87	36.76	158.526	6885
III.2	7.58	44.24	164.005	8875
III.3	4.39	34.27	156.596	7786
III.4	3.84	35.43	155.700	8049
IV.2	3.87	37.53	156.733	6034
IV.3	4.23	38.51	152.977	5253
IV.4	8.79	38.02	147.673	4404
V.4	7.39	30.63	148.974	3170
V.5	5.64	28.82	150.906	2805
VI.5	2.02	34.04	158.040	3914
VI.6	0.85	37.78	157.635	3934
Sum	100.00	--	--	--

Z_s = 90 - 38.02= 51,98° = 0.907 rad

A_z = |147.67 -130|= 17.67°

Spherical angular distance of sky element in the view direction from the sun position χ

$\cos \chi = \cos Z_s \cos Z + \sin Z_s \sin Z \cos A_z$
$\cos \chi = \cos 51.98 \cos 80 + \sin 51.98 \sin 80 \cos 17.67 = 0.8462$
$\chi = 32.20° = 0.562$ rad

4. Calculation of gradation and indicatrix functions :
Gradation function for a sky element in view direction: $\varphi(Z) = 1 + a \exp(b/\cos Z)$
Gradation function for zenith : $\varphi(0°) = 1 + a \exp(b/\cos 0°) = 1 + a \exp b$
Indicatrix function for a sky element in view direction:
$f(\chi) = 1 + c\,[\exp(d\,\chi) - \exp(d\,\pi/2)] + e \cos^2 \chi$
Indicatrix function for zenith : $f(Z_s) = 1 + c\,[\exp(d\,Z_s) - \exp(d\,\pi/2)] + e \cos^2 Z_s$

Tab. E-1.3 Partial parameters

Sky type	Z	Z_s		A_z	$\cos \chi$	χ
	degree	degree	radians	degree	-	radians
II.1	80	59.96	1.046	28.174	0.838	0.576
III.2		45.76	0.799	34.005	0.706	0.787
IV.4		51.98	0.907	17.670	0.846	0.562

For Sky Type IV.4 is :
$\varphi(Z) = 1 - 1 \exp(-0.55 / \cos 80) = 0.9579$
$\varphi(0°) = 1 - 1 \exp(-0.55) = 0.4231$
$f(\chi) = 1 + 10\,[\exp(-3 * 0.562) - \exp(-3\,\pi/2)] + 0.45*0.8462^2 = 3.0847$
$f(Z_s) = 1 + 10\,[\exp(-3 * 0.907) - \exp(-3\,\pi/2)] + 0.45 \cos^2 51.98 = 1.7385$

5. Calculation of relative luminance in the view direction normalised to zenith luminance
$L / L_z = \varphi(Z)\, f(\chi) / ((\varphi(0°)\, f(Z_s))$
IV.4 : $L/ L_z = 0.9579 * 3.0847/(0.4231 * 1.7385) = 4.0175$
6. Calculation of the window luminance on its outer side. Measured zenith luminance data are used to quantify the absolute values of the sky element in view direction
$L_{w,out} = L_{vz}\,(L / L_z)$ IV.4 : $L_{w,out} = 4404* 4.0175 = 17693$ cd/m^2
7. Optical properties of the glazing are defined by its trasmittance which can by calculated as following :
In our case directional angle ψ is the same as view direction elevation γ.
- the partial coefficient of the directional transmittance $\tau_{s\psi} / \tau_{s,nor}$.
$\tau_{s\psi} / \tau_{s,nor} = \cos \psi\,(1+0.5 \sin^2 \psi) = \cos 10°\,(1+ 0.5 \sin^2 10°) = 0.9966.$
- the transmittance coefficient of the normal direction for double transparent glassed window
$\tau_{s,nor} = 0.92 * 0.92 = 0.846$
- the total transmittance $\tau = \tau_{s,nor}\,(\tau_{s\psi} / \tau_{s,nor}) = 0.846 * 0.9966 = 0.8431$
8. Calculation of the luminance level on the inner side of the window glazing is :
$L_{w,in} = \tau * L_{w,out}$
IV.4 : $L_{w,in} = 0.8431 * 17693 = 14917$ cd/m^2, i.e. roughly 15 kcd/m^2.

Results

In a similar way as in case of the sky type IV.4 were derived the resulting critical luminances for other relevant sky types II.1 and III.2 in Table E-1.4.

Tab. E-1.4 Results

Sky type	φ(*Z*)	φ(0°)	f(χ)	f(Z_s)	L / L_z	$L_{w,out}$ cd/m^2	$L_{w,in}$ cd/m^2
II.1	1.11	1.940	1.000	1.000	0.677	3507	2956
III.2	1.00	1.000	1.00	1.487	1.008	8950	7545
IV.4	0.58	0.230	3.85	1.739	4.017	17693	14917

Example 2

Estimate zenith luminance L_z under a clear sky in Athens (φ = 37.97°N, λ_{lo} = 23.72°E, GTM +2) at 10:30 a.m. on the 15th May in 1998 when measured global horizontal illuminance was E_{vg} = 93195 lx and diffuse E_{vd} = 31564 lx.

Solution :

1. From the day angle τ solar declination δ is calculated as follows :
2. Day number on the 15th May is *J* = 135 (February is taken to have 28 days).
3. Day angle $\tau = 2\,\pi\,(J - 1)/365 = 2\,\pi\,(135 - 1)/365 = 2.307$ rad.
4. Solar declination $\delta = 0.006918 - 0.399912\cos(\tau) + 0.070257\sin(\tau) - 0.006758\cos(2\,\tau) + 0.000907\sin(2\,\tau) - 0.002697\cos(3\,\tau) + 0.00148\sin(3\,\tau)$

$$\delta = 0.006918 - 0.399912\cos(2.307) + 0.070257\sin(2.307) - 0.006758\cos(2 * 2.307) + 0.000907\sin(2 * 2.307) - 0.002697\cos(3 * 2.307) + 0.00148\sin(3 * 2.307)$$
$$= 0.3259 \text{ rad} = 18.67\,°.$$

5. To calculate precise values of solar altitude γ_s, i.e. the elevation angle above the horizon, the equation of time as to be applied for deriving true solar time *TST* .
6. Equation of time $ET = 0.17\ \mathrm{SIN}(4\,\pi\,(J\text{-}80)/373)\text{-}0{,}129\ \mathrm{SIN}(2\,\pi\,(J\text{-}8)/355)$

$$ET = 0.17\ \mathrm{SIN}(4\,\pi\,(135\text{-}80)/373)\text{-}0.129\ \mathrm{SIN}(2\,\pi\,(135\text{-}8)/355) = 0.062 \text{ hour.}$$

7. Local time $LT = HOUR + MIN/60 = 10+30/60 = 10.5$ hour.
8. Longitude of the standard meridian is $\lambda_s = 2*15 = 30°$.
9. True solar time $TST=LT+ (\lambda_{lo}\text{-}\lambda_s)/15+ ET = 10.5 + (23.72\text{-}30)/15 +0.062 = 10.14$ hour.
10. Solar hour angle $\zeta = \pi\,TST/12 = 15\,\pi\,10.14406/180 = 2.655709$ rad = 152.16°.
11. Solar altitude $\gamma_s = \arcsin(\sin\varphi \sin\delta - \cos\varphi\cos\delta\cos\zeta)$

$$\gamma_s = \arcsin(\sin(37.97°) \sin 18.67°\text{-} \cos(37.97°) \cos 18.67° \cos 152.16°)$$
$$= 1.030 \text{ rad} = 59.02°\,.$$

12. Linke turbidity factor T_v is applied as an input into the equation for zenith luminance calculation and to express it horizontal extraterrestrial illuminance E_{vo}, mean extinction coefficient a_v and relative optical air mass m has to be used as follows :
13. Extraterrestrial illuminance $E_{vo} = 133800\ (1 + 0.034\cos(2\,\pi\,(J\text{-}2) / 365))$

$$E_{vo} = 133800\ (1 + 0.034\cos(2\,\pi\,(135\text{-}2) / 365)) = 130805 \text{ lx} = 130.805 \text{ klx.}$$

14. Horizontal extraterrestrial illuminance $E_v = E_{vo}\sin(\gamma_s)$=130.805 sin(59.02°) = 112.150 klx.
15. Relative optical air mass $m = 1/(\sin\gamma_s + 0.50572\ (\gamma_s\,° + 6.07995)^{-1.6364})$

$$m = 1/(\sin 59.02° + 0.50572\ (59.02° + 6.07995)^{-1.6364}) = 1.1656.$$

16. Mean extinction coefficient $a_v = 0.1 /(1+ 0.0045\,m) = 0.1 / (1 + 0.0045 * 1.1656) = 0.0995$.
17. Sun horizontal illuminance $E_{vs} = E_{vg} - E_{vd}$ = 93195 - 31564 = 61631 lx = 61.631 klx.
18. Linke turbidity factor $T_v = -\ln(E_{vs}/E_v)/(a_v\,m) = -\ln(61.631/112.150)/(0.0995*1.1656) = 5.16$.
19. After [4] choosing the Sky Standard VI.5 (*A1* = 0.881, *A2* = 0.453, *C* = 4.4 , *D* = 0.79)

$L_z = (A1\ T_v + A2)\ A \sin \gamma_s + 0.7\ (T_v + 1)(\sin \gamma_s)^C / (\cos \gamma_s)^D + 0.04\ T_v = (0.881 * 5.16 + 0.453) \sin 59.02° + 0.7\ (5.16 + 1)\ (\sin 59.02°)^{4.4} / (\cos 59.02°)^{0.79} + 0.04 * 5.16$

$L_z = 8.1997$ kcd/m $^2 = 8200$ cd/m ^{2}.

Table E-3.1

Local time	True solar time	Solar altitude		Horizontal diffuse illuminance
LT	*TST*	γ_s		D_v
hour		deg.	rad	lx
7:30	7.35	4.69	0.0818	1093
8:00	7.85	9.89	0.1727	2299
8:30	8.35	14.83	0.2588	3424
9:00	8.85	19.43	0.3390	4450
9:30	9.35	23.61	0.4121	5359
10:00	9.85	27.29	0.4763	6135
10:30	10.35	30.37	0.5300	6764
11:00	10.85	32.75	0.5715	7238
11:30	11.35	34.33	0.5992	7546
12:00	11.85	35.05	0.6117	7684
12:30	12.35	34.87	0.6086	7649
13:00	12.85	33.80	0.5899	7443
13:30	13.35	31.89	0.5566	7068
14:00	13.85	29.22	0.5100	6532
14:30	14.35	25.89	0.4519	5842
15:00	14.85	22.00	0.3839	5012
15:30	15.35	17.64	0.3079	4054
16:00	15.85	12.90	0.2251	2987
16:30	16.35	7.85	0.1370	1827
17:00	16.85	2.55	0.0445	595

Result

When global horizontal illuminance $E_{vg} = 93195$ lx, diffuse horizontal illuminance $E_{vd} = 31564$ lx and sun height is $\gamma_s = 59.02°$ in Athens the level of zenith luminance $L_z = 8200$ cd/m^2 has to be expected.

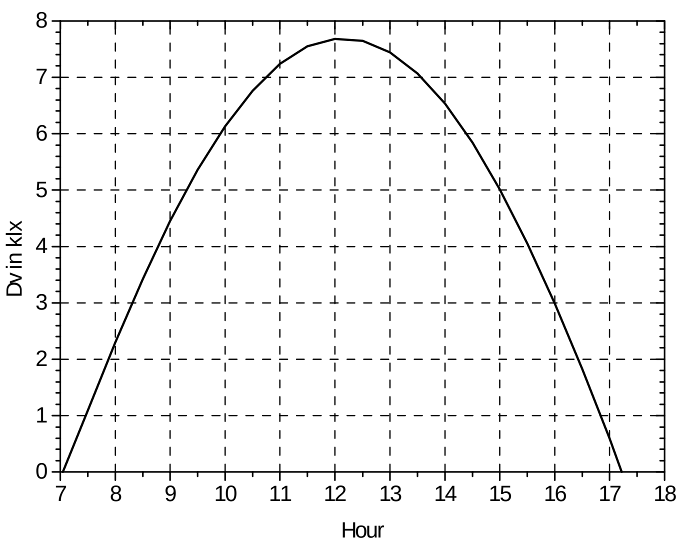
Example 3

Calculate probable horizontal diffuse illuminance D_v under an overcast sky in Athens ($\varphi = 37.97°$N, $\lambda_{lo} = 23.72°$E, GTM +2) during daytime hours on the 10th November in half hour intervals.

Solution :

1. Day number on the 10th November is $J = 314$ (February is taken to have 28 days).
2. Calculate day angle $\tau = 5.388$, solar declination $\delta = -0.2957$ rad. = -16.94 deg. and the equation of time $ET = 0.268$ as in steps 3 to 6 in Example 2.
3. Solar altitude γ_s is calculated as in the steps 7 to 11 in Example 2 for chosen local time *LT*. The values of the true solar time and solar altitude from sunrise to sunset are documented in Table E-3.1.
4. Typical ratio $D_v/E_v = 0.1$ is the most frequent for CIE overcast sky.
5. Probable diffuse horizontal illuminance
$D_v = 133.8 \sin \gamma_s\ D_v/E_v$, i.e. for 7:30 it is $D_v = 133.8 \sin (4.69°)\ 0.1 = 1.0929$ klx = 1093 lx
6. Repeat the same routine for all local times.

Figure E -3.1



Results

Table E-3.1 and Figure E-3.1 display in a table or in a graph results of the daily course of probable diffuse horizontal illuminance D_v on the10th November in Athens.

Table E-4.1

Local Time LT	Horizontal diffuse illuminance In exterior D_v	in interior: Sky component $D_{v, cs}$	Internally reflected component $D_{v, ci}$	Externally reflected component $D_{v, ce}$	Summary in interior D_{vi}
hour	lx				
7:30	1093	13	3	0	16
8:00	2299	27	6	1	34
8:30	3424	41	9	1	51
9:00	4450	53	12	2	67
9:30	5359	64	14	2	80
10:00	6135	73	17	2	92
10:30	6764	80	18	3	101
11:00	7238	86	20	3	109
11:30	7546	90	20	3	113
12:00	7684	91	21	3	115
12:30	7649	91	21	3	115
13:00	7443	89	20	3	112
13:30	7068	84	19	3	106
14:00	6532	77	18	3	98
14:30	5842	69	16	3	88
15:00	5012	59	14	2	75
15:30	4054	48	11	2	61
16:00	2987	36	8	1	45
16:30	1827	22	4	1	27
17:00	595	7	2	0	9

Example 4

Evaluate daylighting in lx on the horizontal work plane in the office designed for the daylight factor DF=1.5 % during the whole day on the 10th November in Athens. The sky component DF_s is taken 79% of DF. internally reflected component DF_i is taken 18% of DF and externally reflected component DF_e is taken 3 % of DF.

Solution :

1. As in example 3 calculate day number J = 314. day angle τ = 5.3880, solar declination δ = -0.2957 rad = - 16.94 deg. and the equation of time ET = 0.268, solar altitude γ_s and horizontal diffuse illuminance D_v defining exterior conditions .
2. Calculate daylighting in the interior in half hour intervals and least in Table E -4.1, e.g. at 7:30

$$D_{vi} = DF * D_v = 0.015 * 1093 = 16.39 \text{ lx}.$$

3. Because $DF = DF_s + DF_i + DF_e$ calculate :

$D_{v,cs}$ = 0.79 D_{vi} = 0.79*16.39 = 12.95 lx, $D_{v,ci}$ = 0.18 D_{vi} = 0.18*16.39 = 2.95 lx

and $D_{v,ce}$ = 0.03 D_{vi} = 0.03*16.39 = 0.49 lx.

4. Continue procedure in step 3 for all local times.

Results

Table E - 4.1 documents changes of daylighting during an overcast day on the 10th November in Athens on the desk in an office which was designed for the daylight factor DF = 1.5 %.

Example 5

Calculate probably diffuse D_v. direct (sun) P_v and and global G_v horizontal illuminance under a clear sky in Bratislava (φ = 48.17°N, λ_{lo} = 17.06°E, GTM +1) during whole day on the 6th August.

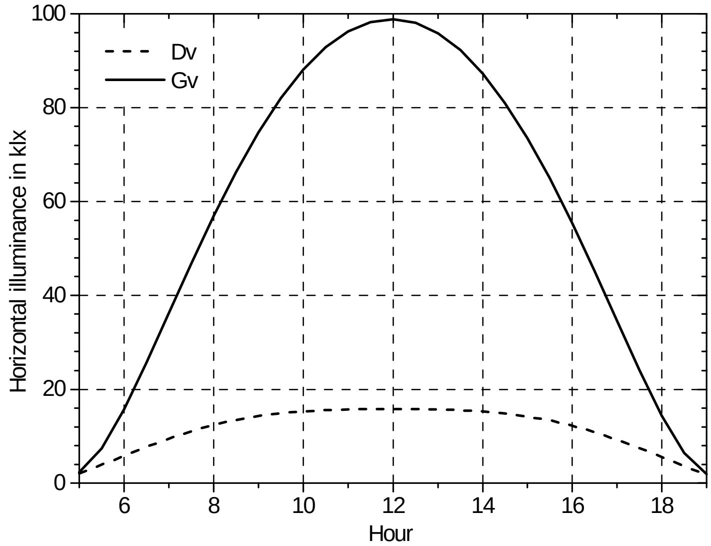
Table E-5.1

Local time *LT*	Solar altitude γ_s	Zenith Luminance L_z	Horizontal illuminance Diffuse D_v	Sun P_v	Global G_v
Hour	deg	cd m^{-2}	klx		
5:00	3.32	0.291	2.042	0.272	2.314
5:30	8.04	0.562	3.939	3.473	7.411
6:00	12.90	0.840	5.851	9.867	15.719
6:30	17.84	1.125	7.722	17.954	25.675
7:00	22.84	1.420	9.481	26.777	36.258
7:30	27.83	1.733	11.057	35.772	46.830
8:00	32.78	2.070	12.397	44.560	56.958
8:30	37.61	2.441	13.475	52.861	66.336
9:00	42.26	2.847	14.295	60.454	74.749
9:30	46.63	3.286	14.889	67.156	82.045
10:00	50.59	3.743	15.297	72.819	88.116
10:30	53.97	4.186	15.560	77.323	92.883
11:00	56.59	4.567	15.716	80.576	96.291
11:30	58.24	4.826	15.793	82.511	98.305
12:00	58.75	4.909	15.814	83.091	98.906
12:30	58.06	4.797	15.786	82.305	98.090
13:00	56.26	4.516	15.698	80.166	95.864
13:30	53.51	4.121	15.528	76.720	92.248
14:00	50.02	3.674	15.245	72.033	87.279
14:30	45.99	3.218	14.812	66.205	81.017
15:00	41.57	2.783	14.186	59.357	73.544
15:30	36.89	2.382	13.328	51.645	64.974
16:00	32.03	2.017	12.211	43.255	55.466
16:30	27.08	1.684	10.833	34.416	45.248
17:00	22.08	1.374	9.224	25.419	34.644
17:30	17.09	1.081	7.444	16.664	24.108
18:00	12.16	0.797	5.563	8.759	14.322
18:30	7.32	0.520	3.649	2.743	6.392
19:00	2.62	0.251	1.759	0.125	1.884

<u>Solution :</u>

1. From the day angle τ solar declination δ is calculated as follows :
2. Day number on the 6th August is $J = 218$ (February is taken to have 28 days).
3. As in example 3 calculate day number $J = 218$, day angle $\tau = 3.7355$ rad and solar declination $\delta = 0.2953$ rad = 16.92 deg. and the equation of time $ET = -0.0995$ hour, solar altitude γ_s and horizontal diffuse illuminance D_v defining exterior conditions.
4. Calculate True solar time *TST* in half hour intervals and least in Table E-5.1, e.g. at 10:30.
5. Local time $LT = HOUR + MIN/60 =$
 $10+30/60 = 10.5$ hour.
6. Longitude of the standard meridian is
 $\gamma_s = 1*15 = 15°$.
7. True solar time
 $TST = LT + (\lambda_{lo}-\lambda_s)/15 + ET = 10.5 + (17.06-15)/15 - 0.0995 = 10.54$ hour.
8. Solar hour angle $\zeta = \pi\ TST/12 = 15\pi\ 10.54/180 = 2.7588$rad $= 158.06°$.
9. Latitude in Bratislava
 $\varphi = 48.17° = 0.8407$ rad.
10. Solar altitude
 $\gamma_s = \arcsin(\sin\varphi \sin\delta - \cos\varphi \cos\delta \cos\zeta)$
 $\gamma_s = \arcsin(\sin(48.17°)\sin 16.92° - \cos(48.17°)\cos 16.92° \cos 158.06°)) = 0.9420$ rad $= 53.97°$.
11. Express extraterrestrial illuminance
 $E_{vo} = 129.986$ klx.
12. Calculate horizontal extraterrestrial illuminance $E_v = 105.127$ klx. relative optical air mass $m = 1.2355$ and mean extinction coefficient $a_v = 0.0994$ in half hour intervals and least. example is at 10:30.
13. The luminance distribution under clear day is representative by Sky Standard V.4 with
 $T_v = 2.5$, [2], [3].
14. Sun horizontal illuminance at 10:30 is $P_v = E_v * \exp(-a_v\ m\ T_v)$
 $P_v = 105.127 \exp(-0.0994*1.2355*2.5) = 77.323$ klx.
15. Choose parameters $A1 = 1.036$, $A2 = 0.71$, $B = 23$, $C = 4.43$, $D = 0.74$, $E = 18.52$ for Sky Standard V.4, [2], [3].
16. When $L_z = [B(\sin\gamma_s)^C/(\cos\gamma_s)^D + E\sin\gamma_s]\ D_v / E_v = Y * D_v / E_v$ then
 $Y = B(\sin\gamma_s)^C / (\cos\gamma_s)^D + E\sin\gamma_s$ [3] and
 $Y = 23(\sin 53.97°)^{4.43} / (\cos 53.97°)^{0.74} + 18.52 \sin 53.97° = 28.2803$.

Figure E-5.1



17. Zenith luminance $L_z = (A1\ T_v + A2)\ \sin\gamma_s + 0.7\ (T_v + 1)\ (\sin\gamma_s)^{C} / (\cos\gamma_s)^{D} + 0.04\ T_v$

$L_z = (1.036*2.5 + 0.71) + 0.7\ (2.5 + 1)\ (\sin 53.97°)^{4.43} / (\cos 53.97°)^{0.74} + 0.04*2.5$

$L_z = 4.186$ cd m^{-2}.

18. If $L_z = Y * D_v / E_v$ then

$D_v = L_z * E_v / Y =$ 4.186*105.127/28.2803 = 15.56 klx

19. Global horizontal illuminance

$G_v = D_v + P_v$=15.56+ 77.323 G_v = 92.883 klx.

20. Continue procedure in step from 5 to 18 for all local times.

Table E-6.1 Selected measured data on the 16th July 1993, a. m.

Time	γ_s	E_{vg}	E_{vd}	L_{vz}	Time	γ_s	E_{vg}	E_{vd}	L_{vz}
Hour	deg	lx	lx	cd/m^2	Hour	deg	lx	lx	cd/m^2
6.10	9.10	8503	7078	784	8.15	33.10	53209	16897	2283
6.20	10.90	10839	8231	878	8.20	34.10	55528	17064	2336
6.25	11.90	12214	8894	960	8.25	35.10	56837	17434	2553
6.30	12.80	13589	9452	1067	8.30	36.00	59504	17471	2550
6.35	13.70	15129	9960	1114	8.35	37.00	60729	17666	2628
6.40	14.70	17117	10563	1167	8.40	38.00	63197	17538	2666
6.45	15.60	18757	11007	1255	8.45	39.00	64395	17833	2710
6.50	16.60	19999	11505	1306	8.50	40.00	65748	18010	2829
6.55	17.50	21042	11936	1397	8.55	41.00	66874	18310	2930
7.05	19.50	25200	13034	1444	9.00	41.90	65864	18846	3106
7.10	20.40	27370	13527	1560	9.05	42.90	68928	19392	3366
7.15	21.40	29788	13876	1651	9.10	43.90	70220	19708	3571
7.20	22.30	31461	14116	1676	9.15	44.90	71844	19771	3618
7.25	23.30	33780	14519	1745	9.20	45.90	71777	19937	3659
7.30	24.30	35403	14733	1767	9.25	46.80	73401	20309	3806
7.35	25.30	38037	15034	1830	9.35	48.80	74792	20952	4318
7.40	26.20	40257	15359	1890	9.40	49.70	78613	20920	4416
7.45	27.20	42377	15564	1997	9.45	50.70	80026	20756	4626
7.50	28.20	44000	15792	2022	10.45	61.80	95646	20904	6637
7.55	29.20	46313	16218	2072	10.50	62.60	96242	20896	7177
8.00	30.10	48472	16484	2176	10.55	63.50	97252	20971	7652
8.05	31.10	49764	16694	2239					

Results

Table E-5.1 and Figure E-5.1 document in a table or in a graph expected changes of the diffuse, direct and global illuminance daylighting during a clear day on the 6th August in Bratislava.

Example 6

Compare 5 - minute measured data during a sunny day on the 16th July 1993 in Athens with the calculated course based on Standard Sky VI.5. Available data are documented in Table E-6.1. and Table E-6.2.

Methodology :

1. Because $\frac{L}{L_z} = \frac{\varphi(Z)\ f(\chi)}{\varphi(0°)\ f(Z_s)}$ then $L = L_z \frac{\varphi(Z)\ f(\chi)}{\varphi(0°)\ f(Z_s)}$ (E-6.1)

Table E-6.2 Selected measured data on the 16th July 1993, p. m.

Time	γ_s	E_{vg}	E_{vd}	L_{vz}
Hour	deg	lx	lx	cd/m^2
14.15	62.10	96424	25184	8902
14.20	61.30	95298	25256	8390
14.30	59.50	94702	24876	7391
14.35	58.60	92002	24944	7315
14.40	57.70	91505	24845	7039
14.45	56.80	89666	24584	6662
14.55	54.90	88225	23614	5914
15.00	54.00	86771	23220	5619
15.05	53.00	85277	23125	5525
15.10	52.10	83819	23067	5399
15.20	50.20	82974	22394	5116
15.25	49.20	81550	22079	4777
15.30	48.20	79529	21584	4626
15.35	47.20	78734	21194	4334
15.40	46.30	76879	20889	4142
15.45	45.30	75802	20777	3995
16.00	42.40	71198	19355	3464
16.05	41.40	70303	18908	3319
16.15	39.40	67438	18045	2980
16.20	38.40	65351	17734	2823
16.30	36.50	62436	17031	2638
16.35	35.50	60762	16657	2512
16.55	31.50	53607	15610	2166
17.05	29.60	50195	14901	1953
17.10	28.60	48058	14603	1874
17.15	27.60	45756	14144	1808
17.20	26.60	44629	13921	1786
17.35	23.70	38931	12889	1582
17.40	22.70	37126	12435	1459
17.45	21.80	35409	12114	1412
17.55	19.80	31395	11394	1331
18.00	18.90	29423	10916	1224
18.05	17.90	27469	10557	1246
18.10	17.00	25929	10240	1192
18.15	16.00	24239	9817	1139
18.20	15.10	22367	9500	1051
18.25	14.10	20562	9212	1054
18.30	13.20	19287	8859	963
18.35	12.30	17465	8439	910
18.40	11.30	15408	8028	888
18.45	10.40	13423	7578	847
18.50	9.50	12115	7111	793
18.55	8.60	10425	6474	677

2. Horizontal illuminace is calculated after

$$D_v = \int_{Z=0}^{\pi/2} \int_{A=0}^{2\pi} L \cos Z \, d\omega = \int_{Z=0}^{\pi/2} \int_{A=0}^{2\pi} L \sin Z \cos Z dZ dA \qquad \text{(E-6.2)}$$

after putting (E-6.1) into (E-6.2) is

$$D_v = L_z \int_{Z=0}^{\pi/2} \int_{A=0}^{2\pi} \frac{\varphi(Z)\ f(\chi)}{\varphi(0°)\ f(Z_s)} \sin Z \cos Z \, dZ \, dA \qquad \text{(E-6.3)}$$

where $f(\chi) = 1 + c\,[\exp(d\,\chi) - \exp(d\,\pi/2)] + e\cos^2\chi$.
$f(Z_s) = 1 + c\,[\exp(d\,Z_s) - \exp(d\,\pi/2)] + e\cos^2 Z_s$.
$\varphi(Z) = 1 + a\exp(b/\cos Z)$.
$\varphi(0°) = 1 + a\exp(b/\cos 0°) = 1 + a\exp b$.
Z_s - Sun zenith angle.
Z - zenith angle of the sky element.
χ - spherical angular distance of the sky element from the sun position.
a, b, c, d, e - gradation and idicatrix parameters.

3. Numerical solution of the double integral (E-6.3) is a summation in between the azimuth A = 0° - 360° and zenith angles Z = 0° - 90°. When the azimuthal and zenith angle steps are taken 5° then

$$D_v = L_z \sum_{Z=0°}^{90°} \sum_{n=1}^{72} \frac{\varphi(Z)\ f(\chi)}{\varphi(0°)\ f(Z_s)} \pi \frac{\sin^2 Z_2 - \sin^2 Z_1}{n} \qquad \text{(E-6.4)}$$

where n = 360°/5° = 72.

4. Zenith luminance L_z is calculated after the following formula :

$$L_z = (A1\ T_v + A2)\ A \sin\gamma_s + 0.7\ (T_v + 1)(\sin\gamma_s)^C / (\cos\gamma_s)^D + 0.04\ T_v \qquad \text{(E-6.5)}$$

where γ_s - solar altitude, $\gamma_s = 90° - Z_s$.

<u>Routine of the calculation</u>

1. Input measured data are : Year, Month, Day, Hour, Min, E_{vg}, E_{vd} and L_{vz}.
2. Calculate horizontal extraterrestrial illuminance E_v, solar altitude γ_s and Linke turbidity factor T_v for each data,

procedure is described in Example 2.
3. Choose gradation and idicatrix parameters a = -1,
b = -0.15, c = 16, d = -3, e = 0.3 and zenith luminance parameters $A1$ = 0.881, $A2$ = 0.453, B = 25.54, C = 4.4, D= 0.79, E = 14.56 for Sky Standard VI.5.

4. Calculate horizontal illuminance D_v using the integration of luminance distribution after sky type selection data as shown above. Use L_z / D_v criteria for the identification of the Sky type, i.e. calculate the ratio L_{vz} /E_{vd} and compare it with values in the range ± 5 % of the theoretical L_z/D_v curve.

{Teoretical standard curve}
Lz/Dv =(1/133.8) (B exp(C ln(sin γs)) / (sin γs exp(D ln(cos γs))) +E)

{Zenith luminance}
Lz = (A1 Tv+A2) sin γs + 0.7(*T*v + 1) exp(C ln(sin γs))/exp(D ln(cos γs)) + 0.04 Tv [cd/m ^{2}]
{ L_z/D_v identification condition}

IF (0.95 Lz/Dv ≤ Lvz/Evd) AND (Lvz/Evd < 1.05 Lz/Dv) then

Zs = 90 - γs SumLe = 0
WHILE Z ≤ 87.5 { Cycle of the zenith angle}
Z1 = Z - 2.5 Z2 = Z + 2.5

{Projetion of the sky element on the horizontal plane}
Δ = π (sin Z2 sin Z2 - sin Z1 sin Z1) / 72

A = 2.5
WHILE A ≤ 177.5 {Start of azimuthal cycle from thesolar meridian}

{Spherical angular distance of the sky element from the sun position}
cos χ = cos Zs cos Z + sin Zs sin Z cos A

IF cos χ =1 THEN χ = 0
ENDIF
IF cos χ <1 THEN χ = PI/2-ArcTan(cos χ /√(1- cos χ cos χ) [rad]
ENDIF

φ(Z) = 1 + a exp (b/cos Z) {Gradation functions}
φ(0°) = 1 + a exp (b/cos 0°) = 1 + a exp b
f(χ) = 1 + c [exp(d χ) - exp(d π/2)] + e cos^2 χ {Indikatrix functions }
f(Zs) = 1 + c [exp(d Zs) - exp(d π/2)] + e cos^2 Zs

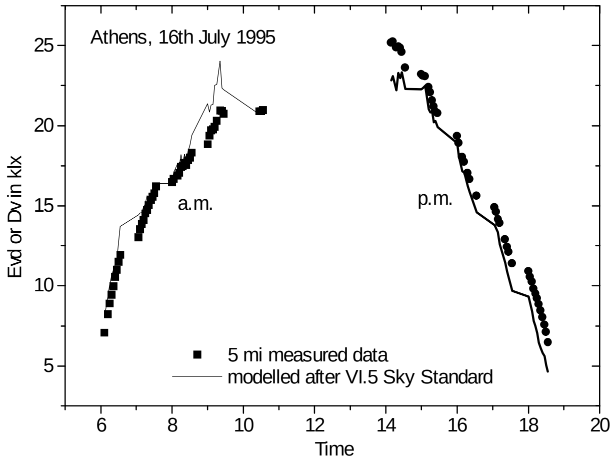
LeToLz = f(χ) φ(Z) / ((φ(0°) f(Zs)) {The relative luminance of the sky element }

SumLe=SumLe+LeToLz Δ {Summation of the relative luminance for half hemisphere}
A = A + 5
ENDWHILE {End of the azimuthal cycle}

Z = Z + 5
ENDWHILE
{End of the zenith angle cycle}
{Sum of relative luminance for whole hemisphere}
Lz,a = 2 SumLe
{Horizontal diffuse illuminance}
Dv = Lz * Lz,a [klx]

{End of the L_z / D_v identification condition}
ENDIF

Figure E-6.1



Results

Table E-6.3, Figure E-6.1, Figure E-6.2 and Figure E-6.3 represent in table or in graph form the comparison of calculated (D_v and L_z) and measured data (E_{vd} and L_{vz}, Table E-6.1

Table E-6.3 Calculated data

Time	T_v	L_z	D_v	Time	T_v	L_z	D_v	Time	T_v	L_z	D_v
Hour	-	cd/m^2	lx	Hour	-	cd/m^2	lx	Hour	-	cd/m^2	lx
6.10	4.49	878	8085	8.50	3.60	3040	18796	15.45	3.68	3734	19905
6.20	4.44	1006	9276	8.55	3.69	3221	19395	16.00	3.54	3260	18896
6.25	4.48	1091	10052	9.00	4.10	3638	21380	16.05	3.40	3041	18113
6.30	4.46	1153	10610	9.05	3.95	3648	20853	16.15	3.25	2731	17148
6.35	4.36	1199	11005	9.10	4.01	3833	21292	16.20	3.28	2653	17088
6.40	4.22	1236	11303	9.15	3.99	3955	21333	16.30	3.17	2408	16236
6.45	4.15	1284	11700	9.20	4.22	4297	22500	16.35	3.12	2296	15837
6.50	4.31	1403	12717	9.25	4.22	4434	22587	16.55	3.04	1944	14584
6.55	4.47	1520	13709	9.35	4.49	5021	24021	17.05	2.97	1773	13778
7.05	4.32	1625	14429	9.40	4.12	4813	22363	17.10	2.98	1716	13568
7.10	4.20	1652	14553	9.45	4.08	4949	22248	17.15	3.00	1660	13345
7.15	4.04	1670	14569	10.45	3.74	6896	20699	17.20	2.88	1541	12581
7.20	4.02	1728	14928	10.50	3.76	7143	20773	17.35	2.82	1347	11447
7.25	3.93	1767	15095	10.55	3.75	7378	20700	17.40	2.76	1267	10900
7.30	3.95	1853	15625	14.15	4.20	7695	22825	17.45	2.74	1208	10498
7.35	3.80	1864	15506	14.20	4.26	7545	23093	17.55	2.71	1094	9693
7.40	3.72	1893	15549	14.30	4.05	6767	22203	18.00	2.71	1046	9335
7.45	3.66	1943	15721	14.35	4.28	6860	23292	18.05	2.69	990	8903
7.50	3.70	2037	16220	14.40	4.21	6539	22965	18.10	2.64	929	8406
7.55	3.66	2093	16384	14.45	4.28	6424	23323	18.15	2.56	859	7814
8.00	3.59	2129	16397	14.55	4.06	5730	22284	18.20	2.58	821	7503
8.05	3.67	2255	17051	15.00	4.06	5545	22264	18.25	2.57	770	7062
8.15	3.67	2424	17614	15.05	4.08	5375	22352	18.30	2.47	702	6452
8.20	3.59	2464	17531	15.10	4.12	5245	22496	18.35	2.48	663	6105
8.25	3.69	2614	18194	15.20	3.83	4600	21026	18.40	2.53	629	5798
8.30	3.52	2589	17658	15.25	3.80	4412	20833	18.45	2.63	607	5599
8.35	3.60	2735	18221	15.30	3.82	4279	20868	18.50	2.54	547	5038
8.40	3.46	2737	17802	15.35	3.70	4014	20198	18.55	2.55	506	4650
8.45	3.55	2899	18392	15.40	3.74	3922	20287				

Figure E-6.2

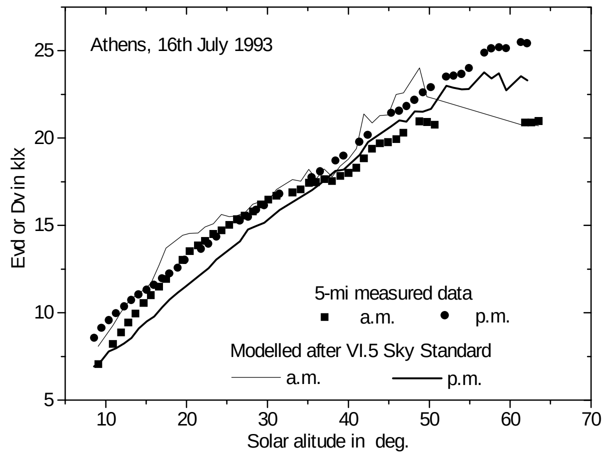
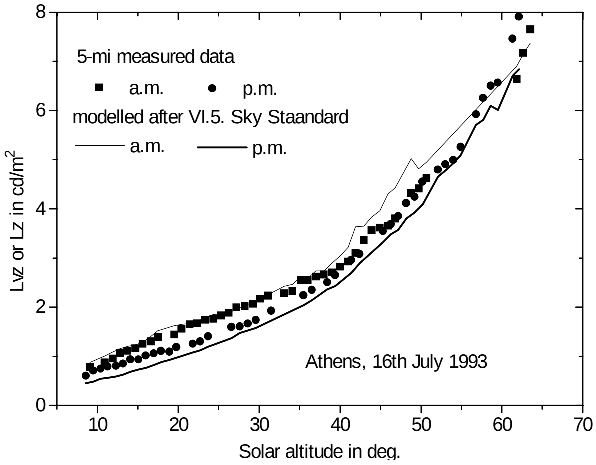
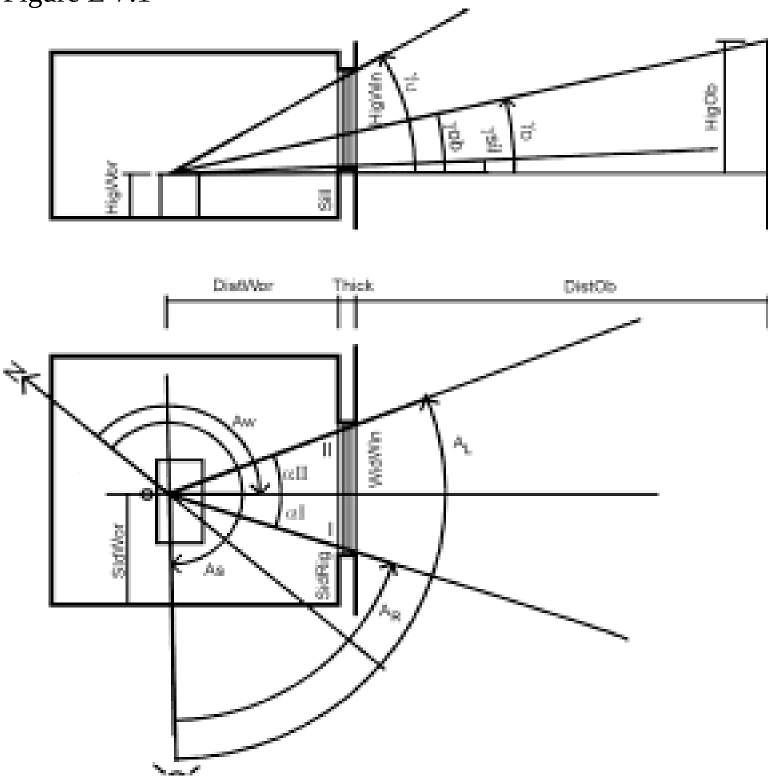


Figure E-6.3



and Table E-6.2) during the morning and the afternoon on the 16th July 1993 in Athens.

Figure E-7.1



Example 7

Compute skylight illuminance in the office during an clear day on the 12th April in Athens. at 14:15. The Sky Standard V.5 which is defined by parameters *a* = -1, *b* = -0.32, *c* = 16, *d* = -3, *e* = 0.3, *A1* = 1.244, *A2* = -0.84, *B* = 27.45, *C* = 4.61, *D* = 0.76, *E* = 16.59 with a typical turbidity factor T_v = 5 is assumed. The azimutal orientation of the window is A_W = 140°. The office window is *HigWin* = 1.8 m high, *WidtWin* = 2.4 m wide with a *Sill* = 0.9 m. It is shaded by the opposite building obstructing at a distance *DistOb* = 30 m and with the height of *HigOb* = 8 m. The thickness of the window wall is *Thick* = 0.3 m. The normal glass transmittance is $\tau_{s,nor}$ = 0.627. The working place is at a distance *DistWor* = 3 m from the window, *SidWor* = 2 m from the side wall.and it is *HigWor* = 0.85 m above the floor. The geometrical situation is shown in Figure E-7.1.

Methodology :

1. Because $\frac{L}{L_Z} = \frac{\varphi(Z)\ f(\chi)}{\varphi(0°)\ f(Z_Z)}$ then $L = L_Z \frac{\varphi(Z)\ f(\chi)}{\varphi(0°)\ f(Z_Z)}$ (E-7.1)

where $\varphi(Z) = 1 + a \exp (b/\cos Z)$, {Gradation functions }
$\varphi(0°) = 1 + a \exp (b/\cos 0°) = 1 + a \exp b$,
$f(\chi) = 1 + c\,[\exp(d\,\chi) - \exp(d\,\pi/2)] + e \cos^2 \chi$, {Indikatrix functions }
$f(Z_S) = 1 + c\,[\exp(d\,Z_S) - \exp(d\,\pi/2)] + e \cos^2 Z_S$,
Z_S - Sun zenith angle,
Z - zenith angle of the sky element,
χ - spherical angular distance of the sky element from the sun position,
a, b, c, d, e - gradation and idicatrix parameters,

2. Horizontal illuminace is calculated due to integration over the window solid angle defined by the elevation and azimuthal angles after

$$D_V = \int_{\gamma=0}^{\pi/2} \int_{A=0}^{2\pi} L \sin\gamma\; d\,\omega = \int_{\gamma=0}^{\pi/2} \int_{A=0}^{2\pi} L \sin\gamma \cos\gamma d\,\gamma\; dA \qquad \text{(E-7.2)}$$

3. The window solid angle has to be overlayed by a regular mesh of sky elements defined after
.
Solid angle $d\omega = dS/r = \Delta\gamma\, \Delta A\,/\,r$ [sr]

when $\Delta\,\gamma$ is the elevation difference angle of the sky element in radians.
$\Delta\,A$ is the azimuthal difference angle of the sky element in radians.

4. Putting (E-7.1) into (E-7.2) is

$$D_V = L_Z \int_{\gamma=0}^{\pi/2} \int_{A=0}^{2\pi} \frac{\varphi(Z)\; f\,(\chi)}{\varphi(0°)\; f\,(Z_S)} \sin\gamma \cos\gamma\; d\,\gamma\; dA \qquad \text{(E-7.3)}$$

5. The numerical solution of the double integral (E-7.3) for the glazed window is a summation of sky elements with appropriate luminance distribution within the window border azimuths, i.e. from A_R to A_L and elevations from γ_D to γ_U (Figure E-7.1).
Then the horizontal diffuse illuminance reaching the working point through the unglazed aperture is

$$D_v = L_Z \sum_{\gamma=\gamma d}^{\gamma u} \sum_{A=Ar}^{Al} \frac{\varphi(Z)\; f\,(\chi)}{\varphi(0°)\; f\,(Z_S)} \frac{\cos\gamma_1 - \cos\gamma_2}{2}\, dS \qquad \text{[klx]} \quad \text{(E-7.4)}$$

where zenith luminance for overcast skies is $L_Z = (D_V/E_V)\,(B\,(\sin\gamma_S)^C / (\cos\gamma_S)^D + E\ \sin\gamma_S)$ in cd/m^2.

<u>Calculation procedure :</u>
1. The chosen day is April the 12th, i.e. its number is $J = 102$ (February is taken to have 28 days).
2. Day angle $\tau = 2\,\pi\,(J - 1)/365 = 1.739$ rad.
3. Solar declination $\delta = 0.006918 - 0.399912 \cos(\tau) + 0.070257 \sin(\tau) - 0.006758 \cos(2\,\tau) + 0.000907 \sin(2\,\tau) - 0.002697 \cos(3\,\tau) + 0.00148 \sin(3\,\tau) = 8.393° = 0.146$ rad

4. Equation of time ET = 0.17 sin(4 π (J-80)/373)-0.129 sin(2 π (J-8)/355) = -0.014 hour.
5. Local time $LT = HOUR + MIN/60$ = 14.25 hour.
6. Longitude of the standard meridian for Bratislava is γ_s = 2*15 = 30°.
7. True solar time $TST = LT + (\lambda_{lo} - \lambda_s)/15 + ET$ = 13.818 hour.
8. Solar hour angle $\zeta = \pi\ TST/12$ = 3.617 rad.
9. Solar altitude γ_s = arcsin (sin φ sin δ - cos φ cos δ cos ζ) = 51.54° = 0.9 rad.
10. Sun azimuth A_s = 226.77° = 3.958 rad calculated after the following algorithm :

```
POM = cos ( δ ) (cos ( φ ) SIN( δ ) /cos ( δ ) + sin( δ ) cos (15 π TST/180)) / cos (γs)
IF (0 < TST) AND (TST ≤ 12) THEN
        As = π / 2 - arctan ( POM / √(1 -POM²))
ENDIF
IF 12 < TST THEN
        As = 2 π - (π / 2 - arctan (POM / √(1 - POM²)))
ENDIF
```

11. The critical azimuthal angles for this window have to be taken from the normal of window A_n, i.e. azimuth from sun meridian, to its right A_R and left A_L corner. From the room geometry in Figure E-7.1

$A_n = (A_s - A_w)$ = 226.77 - 140 = 86.77°

α_I = arctan (pom/sqrt(1-pom^2)) = 19.47°

where pom = (SidWor-SidRig)/(DistWor+Thick)

α_{II} = arctan($pom/\sqrt{(1-pom^2)}$) = 23.2°

where $pom = (WidtWin + SidRig - SidWor)/(DistWor + Thick)$

$A_R = A_n - \alpha_I$ = 67.3° = 1.175 rad $\qquad$ $A_L = A_n + \alpha_{II}$ = 109.97° = 1.919 rad.

12. D_v/L_z calculated from window depends on the elemental azimuthal difference dA when the step A_{step} = 2° is choosen.

A_{num} = INT(($A_L - A_R$)/ A_{step}) = INT(109.974 - 67.3) / 2) = 21

where function INT is integer part of the argument

then dA = ($A_L - A_R$)/ A_{num} = (109.974 - 67.3) / 21 = 2.03 ° = 0.035 rad.

13. To calculate the summation of sky elements with appropriate luminance distribution the subroutine LuminanceIntegration has to be called. The resulting D_v/L_z for the window is

DvRel = SumLeLz = 0.0342.

14. For the determination of absolute values zenith luminance has to be calculated using B, C, D and E parameters, i.e. :

$L_z = (A1 * T_v + A2) \sin\gamma_s + 0.7 (T_v + 1) (\sin\gamma_s)^C / (\cos\gamma_s)^D + 0.04\ T_v$ = 6364.1 cd/m2.

15. The horizontal diffuse illuminance D_v on the desk from windows is

$D_v = L_z$ * DvRel = 217.6 lx.

<u>Subroutine :of the LuminanceIntegration.</u>

```
Zs =90 - γs               SumLeLz = 0          A = AR + dA/2
WHILE A ≤ (AL- dA/2)                          {The start of the azimuth cycle }
        {The criterial elevation angles γob - obstruction, γU - upper window frame and γsil -
sill}
        γob = arctan (cos ( | An - A| ) HigOb / (DistWor + Thick + DistOb))
        γU= arctan (cos ( | An - A| ) (HigWin + Sill - HigWor) / (DistWor + Thick))
```

$\gamma sil = \arctan(\cos(|An - A|)\,(Sill\text{-}HigWor)/DistWor)$

IF $\gamma sil < \gamma ob$ THEN $\gamma_D = \gamma ob$ {Testing of elevation angles}
ELSE $\gamma_D = \gamma sil$
ENDIF

$\gamma\_num = INT((\gamma_U - \gamma_D) / \gamma step)$
$d\gamma = ((\gamma_U - \gamma_D) / \gamma\_num$ { The elemental elevation difference $d\gamma$}

$\gamma = \gamma_D + d\gamma/2$
WHILE $\gamma \le \gamma_U - d\gamma/2$ {The start of the elevation cycle}
$\gamma 1 = \gamma - d\gamma/2$ $\gamma 2 = \gamma + d\gamma/2$

{Projection of the sky element in the horizontal plane}
$\Delta 1 = dA\,(\cos\gamma 1\ \cos\gamma 1 - \cos\gamma 2\ \cos\gamma 2) / 2$

{Spherical angular distance χ of sky element from sun position}
$Z = \pi/2 - \gamma$
$\cos\chi = \cos Zs\ \cos Z + \sin Zs\ \sin Z \cos A$
IF $\cos\chi < 1$ THEN $\chi = \pi/2 - \arctan(\cos\chi / \sqrt{(1 - \cos\chi\ \cos\chi)})$
ELSE $\chi = 0$
ENDIF

$\varphi(0°) = 1 + a \exp(b/\cos 0°) = 1 + a \exp b$ {Gradation functions }
$\varphi(Z) = 1 + a \exp(b/\cos Z)$
$f(\chi) = 1 + c\,[\exp(d\,\chi) - \exp(d\,\pi/2)] + e \cos^2\chi$ {Indicatrix functions }
$f(Zs) = 1 + c\,[\exp(d\,Zs) - \exp(d\,\pi/2)] + e \cos^2 Zs$

$LeToLz = f(\chi)\varphi(Z)/((\varphi(0°)\ f(Zs))$ {Relative luminance on the outer side of window}

$\cos\psi = \cos\gamma \cos|An - A|$ $\sin^2\psi = 1 - \cos^2\psi$ { The angle of incidence}
{ Partial coefficient of the directional transmitance}
$\tau s\psi/\tau s,nor = \cos\psi\,(1+0.5 \sin^2\psi)$
$\tau = \tau s,nor\,(\tau s\psi/\tau s,nor)$ { Total transmitance}

$SumLe = SumLe + \tau * \Delta 1 * LeToLz$ {Sumation of the relative luminance }

$\gamma = \gamma + d\gamma/2$
ENDWHILE {The end of the elevation cycle}

$A = A + dA$
ENDWHILE {End of the azimuthal cycle }

Results

On the desk in the office orientated 140° from the North under clear sky conditions on the 12th April at 14:15 in Athens can be expected skylight illuminance D_v = 217 lx.

Example 8

Calculate skylight illuminance in the office during a overcast day on the 12th April in Brauslava at 14:15. The Sky Standard I.1 which is defined by parameters a = 4, b = -0.7, c = 0, d = -1, e = 0, B = 54,63, C = 1, D = 0, E = 0 with a typical ratio D_v/E_v = 0.1. The azimuthal orientation of the window, glazing and geometrical configuration is the same as in Example 7.

Table E-8.1

Step in Example 2	Describtion	Symbol	Value	Unit
2	Day number	J	102	
3	Day angle	τ	1.739	rad
4	Solar declination	δ	8.393	deg
6	Equation of time	ET	-0.014	hour
7	Local time	LT	14.25	hour
8	Longitude of the standard meridian	λ_s	15.00	deg
9	True solar time	TST	14.374	hour
10	Solar hour angle	ζ	3.763	rad
11	Solar altitude	γ_s	40.18	deg
Step in Example 7	Describtion	Symbol	Value	Unit
10	Sun azimuth As	A_s	228.96	deg
13	Relative diffuse illuminance	DvRel	0.0209	

Solution :

1. As in Example 7 calculated parameters are summarised in Table E-8.1 except zenith luminance L_z under the clear sky I.1 that has to be calculated.
2. After [2] choosing the Sky Standard I.1 zenith luminance is

$$L_z = (A1\ T_v + A2)\ \sin\gamma_s + 0{,}7\ (T_v + 1)\ (\sin\gamma_s)^C / (\cos\gamma_s)^D + 0.04\ T_v = 3524.67\ \text{cd/m}^2$$

3. 15. The horizontal diffuse illuminance D_v on the working place is

$D_v = L_z$ * DvRel = 73.627 lx.

Results

On the desk in the office orientated 140° from the North under overcast conditions on the 12th April at 14:15 in Bratislava can be expected skylight illuminance D_v = 73 lx.

Example 9

Find out the sky component of daylight factor DF_s under the same conditions as in the Example 7.

Solution :

1. Calculate γ_s = 40.180° using step 9 in Example 7, L_z = 3524.67 cd/m^2 (step 14) when parameters a = 4, b = 0.7, c = 0, d = -1, e = 0, B = 54.63, C = 1, D = 0, E = 0 are defined for Sky Standard I.1.
2. As in previous example the interior horizontal diffuse illuminance D_{vs} = 73 lx by the summation of illuminance from window (step 15).
3. To calculate sky component of daylight factor DF_s exterior horizontal illuminace from whole sky vault D_v is needed which is determined after the following algorithm :

Zs =90 - γs Z = 2.5° SumLe = 0

WHILE Z ≤ 87.5 {The start of the zenith angle cycle}
Z1 = Z - 2.5 Z2 = Z + 2.5

Δ = π / 72 (cos Z1 cos Z1 - cos Z2 cos Z) / 72

Ael = 2.5
WHILE Ael ≤ 177.5 do begin {The start of the azimuth cycle }

{Spherical angular distance χ of sky element from sun position}
cos χ = cos Zs cos Z + sin Zs sin Z cos Ael
IF cos χ =1 THEN χ = 0
ENDIF
IF cos χ <1 THEN χ = π/2 - arctan(cos χ /√(1- cos χ cos χ))
ENDIF

φ(0°) = 1 + a exp (b/cos 0°) = 1 + a exp b {Gradation functions}
φ(Z) = 1 + a exp (b/cos Z)
f(χ) = 1 + c [exp(d χ) - exp(d π/2)] + e $\cos^2$ χ {Indicatrix functions}
f(Zs) = 1 + c [exp(d Zs) - exp(d π/2)] + e $\cos^2$ Zs

Le = f(χ) * φ(Z) {Relative luminance }

SumLe = SumLe + Δ* Le {Sumation of the relative luminance}

Ael = Ael + 5
ENDWHILE {End of azimuthal cycle}

Z = Z + 5
ENDWHILE {The end of zenith angle cycle}

DvRel = 2 SumLe
LzRel = f(Zs) * φ(Z)
LzToDv = LzRel / DvRel {The Sky Standard selection parameter}

Dv = Lz / LzToDv {Diffuse horizontal illuminance}

4. As DvRel = 7.3138, LzRel = 2.9863 and L_z / D_v = LzToDv = 0.4083,
finaly is the exterior horizontal illuminace

$D_v = L_z / (L_z / D_v) = 3524.67 / 0.4083 = 8.632$ klx = 8632 lx.

5. Alternatively if L_z / D_v is known for a particular Sky Standard after the formula

$$L_z / D_v = (B\,(\sin \gamma_s)^C / ((\sin \gamma_s)\,(\cos \gamma_s)^D + E)) / E_{vo} =$$

$$54.63\,(\sin 40.180°)^1 / ((\sin 40.180°) * (\cos 40.180°)^0) + 0) / 133.8$$

$$L_z / D_v = 54.63/133.8 = 0.4083$$

where E_{vo} is the luminous extraterrestrial solar constant E_{vo} = 133.8 klx.

then $D_v = L_z / (L_z / D_v)$ = 3524.67 /0.4083 = 8.632 klx.

6. Because the sky component of daylight factor is the ratio of interior to exterior sky illuminance in percent

$$DF_s = 100\ D_{vs} / D_v = 100 * 73 / 8632 = 0.846\ \%$$

Results
Under an overcast sky on the 12th April at 14:15 in Bratislava the sky component of the Daylight Factor in the chosen point of the investigated office is DF_s = 0.846 %. which means that in the working place there is 0.846 % of the exterior horizontal illuminance.

Example 10

Determine the percentage occurrence of all Sky Standards in the winter season at Bratislava using IDMP daylight 5-minute data measured during the 5 - year period 1994 - 1998.

Methodology :
During the winter season (November, December, January and February) sunheight reaches to maximun γ_s = 33.8°. The theoretical L_z/D_v changes for all Sky Standards form fluent curves in the region γ_s = 5° - 35°. In this case it is advantageous to use the method MAC (Mean of Adjacent Curves) for evaluating all measured data after passing the CIE Quality Contol tests after [6]. The range of each sky type is defined by two mean L_z/ D_v values of adjacent curves of Sky Standard respecting the solar altitude dependency. It is usefull to sort the data after sunshine duration which provides information on daylight sunny conditions. The criteria for the determination of sunny situations after [6] is $E_{es} \geq 120$ W/m^2 and for situation without sun $E_{es} < 120$ W/m^2 where E_{es} is the measured direct irradiance. The percentage occurrence of different Sky Standards is calculated after summarisation of cases in both solar situations.

Routine of the calculation :
1. The declaration of variables part of the program.
2. The program part in which the continual reading of all data from choosen dataset, i.e. Month *Mon*, Day *Day*, Year *Y*, Hour *Hour*, Minute *Min*, global horizontal illuminance E_{vg}, diffuse horizontal illuminance E_{vg}, global horizontal irradiance E_{eg}, diffuse horizontal irradiance E_{ed} and zenith luminance L_{vz} is processed respecting own data formatting .
3. The program part in which solar altitude γs is calculated, e.g. as in Example 7 step 9 .
4. The routine of percentage occurrence of each Sky Standard follows commands :

```
IF (5<γs) and (γs ≤ 35) THEN                      {The start of solar altitude cycle }

Evo = 133.800 (1 + 0.034 cos(2 π (135-2) / 365))   {Normal extraterrestrial illuminance}
Ev = Evo sin(γs )                                  {Horizontal extraterrestrial illuminance}
LvzToEvd = Lvz/Evd                                 {Ratio of measured Lvz/Evd }
Ees = (Eeg - Eed)/sin(π γs /180)                   {Direct irradiance}

     IF Evg ≤ 1.2 Ev THEN                          {The quality control test}
     IF Evd ≤ 0.8 Ev THEN                          {The quality control test}
```

```
        IF Evd ≤ 1.1 Evg THEN                                   {The quality control test}

                                        {The Sky Standards condition of the Lvz/Evd range}
IF (0.05 < Lvz/Evd) and (Lvz/Evd < 0.6) THEN

                        {Theoretical curves for the selection ratio Lz/Dv for each Sky Standard}
FOR X = 1 TO 15     LzToDv[X]  = (B[X] (sin γs)^C[X] / (cos γs)^D[X]  + E[X] sin γs) /Evo
ENDFOR

FOR SSLD = 1 TO 15                                      {The start of the Sky Standard cycle}
        IF SSLD = 1 THEN                UpBord = 0.6
                END ELSE                UpBord = DowBord
        ENDIF
        IF SSLD = 15 THEN DowBord = 0.05
                END ELSE        DowBord = ( LzToDv[SSLD]+ LzToDv[SSLD+1])/2
        ENDIF

        IF Ees < 120  THEN                          {The condition for skies without sunshine }
                                                        {Summation  of Sky Standard cases }
                IF (DowBord ≤ LvzToEvd) AND (LvzToEvd < UpBord) THEN
                        NumCaseOv120[SSLD] = NumCaseOv120[SSLD] + 1
                ENDIF
        ENDIF                                               {The end of skies without sunshine }

        IF 120  ≤ Ees  THEN                            {The condition for skies with sunshine }
                                        {Summation of Sky Standard cases in this group }
                IF (DowBord  ≤  LvzToEvd) AND (LvzToEvDv < UpBord) THEN
                        NumCaseSun120[SSLD]=NumCaseSun120[SSLD] + 1
                ENDIF
        ENDIF                                           {The end of skies with sunshine }
ENDFOR                                                      {The end of Sky Standard cycle }
ENDIF                                   {The end of Rage of the Standards condition}
ENDIF                                               {The end of quality control test}
ENDIF                                                   {The end of quality control test}
ENDIF                                               {The end of quality control test}
ENDIF                                               {The end of solar altitude cycle}

TotalOv120 = 0              TotalSun120 = 0                 {The calculation of total sum}
FOR SSLD = 1 TO 15                  TotalOv120 = TotalOv120 + NumCaseOv120[SSLD]
                            TotalSun120 = TotalSun120 + NumCaseSun120[SSLD]
ENDFOR

                            {The calculation of occurence percentage of Sky Standard }
SumPercOv120 = 0    SumPercSun120 = 0
FOR SSLD = 1 TO 15                          {The cycle start of the particular Sky Standard }
        writeln(SSLD,  NumCaseOv120[SSLD],  100*NumCaseOv120[SSLD]  /  TotalOv120)
        writeln(NumCaseSun120[SSLD], 100*NumCaseSun120[SSLD] / TotalSun120)
```

SumPercOv120 = SumPercOv120 + 100 NumCaseOv120[SSLD] / TotalOv120
SumPercSun120 = SumPercSun120 + 100 NumCaseSun120[SSLD] / TotalSun120
ENDFOR {The cycle end of the Sky Standard }

Table E-10.1

Sky Standard		Winter season(I+II+XI +XII)					
		Number of cases	Percentage	Number of cases	Percentage	Number of cases	Percentage
Code	No.	without sun	%	with sun	%	Sum	%
1	2	3	4	5	6	7	8
I.1	1	6998	17,83	62	0,39	7060	12,82
I.2	2	7289	18,57	39	0,25	7328	13,31
II.1	3	6530	16,64	56	0,35	6586	11,96
II.2	4	4142	10,55	91	0,58	4233	7,69
III.1	5	3146	8,02	120	0,76	3266	5,93
III.2	6	3121	7,95	212	1,34	3333	6,05
III.3	7	2398	6,11	453	2,86	2851	5,18
III.4	8	1502	3,83	562	3,55	2064	3,75
IV.2	9	1165	2,97	661	4,18	1826	3,32
IV.3	10	1009	2,57	1033	6,53	2042	3,71
IV.4	11	808	2,06	1877	11,87	2685	4,88
V.4	12	568	1,45	4792	30,30	5360	9,73
V.5	13	316	0,81	3617	22,87	3933	7,14
VI.5	14	129	0,33	1354	8,56	1483	2,69
VI.6	15	124	0,32	887	5,61	1011	1,84
Sum		39245	100,00	15816	100,00	55061	100,00

Note that this table corresponds with columns 3,4,5 and 6 in Table 1 respectively.

writeln('   ', TotalOv120, SumPercOv120, TotalSun120, SumPercSun120)

<u>Results</u>

Bratislava five year IDMP data during the winter seasons present two dominant occurrences of Sky Strandards. The Sky Strandard I.2 is the most occurring under overcast and cloudy situations without sun (18.57 %, Table 10.1) contrary to the Sky Standard V.4 under sunny conditions (30.3 %, column 6). The comparison of all data (column 8 in Table E-10.1) with partial results in column 4 and 6 show various occurrence distributions of all sky standards. While overcast and sunny situations are showing an asymmetrical distribution after their addition two peaks can be observed with the first maximum 13.31 % in I.2 and the second maximum 9.73 % in V.4 Sky Standard.

Example 11

Determine the percentage occurrence of all Sky Standards in the summer season at Bratislava using IDMP 5-minute daylight data measured during the 5 - year period 1994 - 1998.

<u>Methodology :</u>

The summer daylight conditions at Bratislava are represented in a period from May to August. The sunheights exceed 35° and at midday are rising above 60°. The theoretical L_z/D_v changes for all Sky Standards are irregularly intersecting in the region of sunheights above 35°. Because frequency distributions of all Sky Standards are creating a regular surface forming a solid shape it is useful to sort measured data in the defined range around L_z/D_v theoretical curves. In such a case it is advantageous to use the PRC (Percentage Range of Curves) method. The strips around each L_z/D_v curve is defined by the variance from the theoretical curve expressed in percentage of variance ± 1% or ± 2.5%. Furthermore it is inevitable to sort the data after sunshine presence similarly as in Example 10.

Routine of the calculation :

1. The declaration of variables in the program part.
2. The program part in which the continual reading of all data from the choosen dataset is processed, i.e. Month *Mon*, Day *Day*, Year *Y*, Hour *Hour*, Minute *Min*, global horizontal illuminance E_{vg}, diffuse horizontal illuminance E_{vg}, global horizontal irradiance E_{eg}, diffuse horizontal irradiance E_{ed} and zenith luminance L_{vz} respecting own data formatting .
3. The program part in which solar altitude γ_s is calculated, e.g. as in Example 7 step 9 .
4. The routine of percentage occurrence of each Sky Standard follows commands :

```
IF (5 < γs) and (γs ≤ 80) THEN                          {The start of solar altitude cycle }
Evo = 133.800 (1 + 0.034 cos(2* π (135-2) / 365))       {Normal extraterrestrial illuminance}
Ev = Evo sin(γs )                                        {Horizontal extraterrestrial illuminance}
LvzToEvd = Lvz / Evd                                     {Ratio of measured Lvz/Evd }
Ees = (Eeg - Eed)/sin(γs)                                {Direct irradiance}

  IF Evg ≤ 1.2 Ev THEN                                   {The quality control test}
  IF Evd ≤ 0.8 Ev THEN                                   {The quality control test}
  IF Evd ≤ 1.1 Evg THEN                                  {The quality control test}

                                   {The Sky Standards condition of the Lvz/Evd range}
IF (0.05 < Lvz / Evd) and (Lvz / Evd < 0.6) THEN

                     {Theoretical curves for the selection ratio Lz /Dv for each Sky Standard}
FOR X = 1 TO 15        zToDv[X]  = (B[X] (sin γs)^C[X] / (cos γs)^D[X]  + E[X] sin γs) /Evo
ENDFOR

FOR SSLD = 1 TO 15                                       {The start of the Sky Standard cycle}
      IF Ees < 120  THEN                                 {The condition for skies without sunshine }
                                                         {Test of overlapping data in the ± 2.5% strip}
            DowBord = 0.975 LzToDv[SSLD]          UpBord = 1.025 LzToDv[SSLD]
            IF (DowBord ≤ LvzToEvd) AND (LvzToEvd < UpBord) THEN
                  CodeOverlapOv = CodeOverlapOv + 1      SSLD_Ov = SSLD
            ENDIF
      ENDIF                                              {The end of skies without sunshine }
      If 120 ≤ Ees  then begin                           {The condition for skies with sunshine }
            γs_crit = 80
            IF (SSLD = 1) OR (SSLD = 3) THEN                       γs_crit = 45
```

```
                ENDIF
                IF (SSLD = 2) OR (SSLD = 4) OR (SSLD = 5) THEN       γs_crit = 50
                ENDIF
                IF γs < γs_crit THEN                  {Test of overlapping in  the ± 2.5% strip}
                DowBord = 0.975 LzToDv[SSLD]             UpBord = 1.025 LzToDv[SSLD]

                IF (DowBord ≤ LvzToEvd) AND (LvzToEvd < UpBord) THEN
                CodeOverlapSun = CodeOverlapSun + 1               SSLD_Sun = SSLD
                ENDIF
                ENDIF                                          {The end of γs_crit condition }
        ENDIF                                          {The end of skies with sunshine }
ENDFOR                                                    {The end of Sky Standard cycle }

                            {Summation of Sky Standard cases in  the ± 2.5% strip}
        IF CodeOverlapOv = 1 THEN
                NumCaseOv120[SSLD_Ov] = NumCaseOv120[SSLD_Ov] + 1
        ENDIF
        IF CodeOverlapSun = 1 THEN
                NumCaseSun120[SSLD_Sun] = NumCaseSun120[SSLD_Sun] + 1
        ENDIF

CodeOverlapOv = 0   CodeOverlapSun = 0  SSLD_Ov = 99                 SSLD_Sun = 99

ENDIF                               {The range end of the Sky Standard condition}
ENDIF                                              {The end of quality control test}
ENDIF                                              {The end of quality control test}
ENDIF                                              {The end of quality control test}
ENDIF                                              {The end of solar altitude cycle}

                                                         {The calculation of the total sum}
TotalOv120 = 0               TotalSun120 = 0
FOR SSLD = 1 TO 15                   TotalOv120 = TotalOv120 + NumCaseOv120[SSLD]
                             TotalSun120 = TotalSun120 + NumCaseSun120[SSLD]
ENDFOR

                          {The calculation of occurence percentage of Sky Standard }
SumPercOv120 = 0    SumPercSun120 = 0

FOR SSLD = 1 TO 15                                {The cycle start of the printing results}
        writeln(SSLD,  NumCaseOv120[SSLD],   100*NumCaseOv120[SSLD]  /  TotalOv120)
        writeln(NumCaseSun120[SSLD], 100*NumCaseSun120[SSLD] / TotalSun120)

SumPercOv120 = SumPercOv120 + 100*NumCaseOv120[SSLD] / TotalOv120
SumPercSun120 = SumPercSun120 + 100*NumCaseSun120[SSLD] / TotalSun120
ENDFOR                                             {The cycle end of the printing results}

writeln('        ', TotalOv120, SumPercOv120, TotalSun120, SumPercSun120)
```

Table E-11.1

Sky Standard		Summer season V+VI+ VII+VIII)					
		Number of cases	Percentage	Number of cases	Percentage	Number of cases	Percentage
Code	No.	without sun	%	with sun	%	Sum	%
1	2	3	4	5	6	7	8
I.1	1	2554	11,31	16	0,07	2570	5,55
I.2	2	2613	11,57	33	0,14	2646	5,72
II.1	3	3079	13,63	45	0,19	3124	6,75
II.2	4	2745	12,15	94	0,40	2839	6,13
III.1	5	2127	9,42	246	1,04	2373	5,13
III.2	6	2487	11,01	2312	9,76	4799	10,37
III.3	7	2234	9,89	1410	5,95	3644	7,87
III.4	8	1535	6,80	1051	4,43	2586	5,59
IV.2	9	789	3,49	1517	6,40	2306	4,98
IV.3	10	737	3,26	2245	9,47	2982	6,44
IV.4	11	747	3,31	5186	21,88	5933	12,82
V.4	12	479	2,12	5065	21,37	5544	11,98
V.5	13	228	1,01	3113	13,14	3341	7,22
VI.5	14	168	0,74	941	3,97	1109	2,40
VI.6	15	63	0,28	426	1,80	489	1,06
Sum		22585	100,00	23700	100,00	46285	100,00

Results

After Bratislava five year IDMP 5 minute data there have to be expected one peak asymmetrical occurrence distributions of Sky Strandards during the summer seasons. The Sky Strandard II.1 is the most occurring one under overcast and cloudy situations without sun (13.63 % in Table E-11.1) while under sunny conditions the Sky Standards IV.4 and V.4 (21.88 % and 21.37 % respectively) are most frequent. The comparison of all data (column 8 in Table E-11.1) with partial results in column 4 and 6 show dominant occurrence distributions of white - blue skies (Standard IV.4) and the CIE Clear Sky (Standard V.4).

References

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